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ABSTRACT

Five segments of the Self-Paced Physics Course materials are presented in this problems and solutions book for use as the second part of student course work. The subject-matter topics are related to circular motion, work, power, kinetic energy, potential energy, conservative forces, conservation of energy, spring problems, center of mass, and conservation of momentum. Contained in each segment are information panels, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. A study guide is provided for each segment, and used to answer the true-false questions and to reveal directions for reaching solutions when the correct answer is chosen. When the core problem is answered incorrectly, the study guide requires the student to follow the remedial or enabling loop, leading, in turn, to the solution of core-primed questions. Also included is an answer sheet for the five segments. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

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Course Materials - Fall, 1970

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ED 075239

UNITED STATES NAVAL ACADEMY

Self-Paced

PHYSICS

SEGMENTS 6-10

SE 016 068

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PROBLEMS -- AND THEIR CORRECT-ANSWER PAGES

Segments 6, 7, 8, 9 and 10

Segment 6		Segment 7		Segment 8		Segment 9		Segment 10	
P. #	Solution Page	P. #	Solution Page	P. #	Solution Page	P. #	Solution Page	P. #	Solution Page
1	13a	1	26b	1	16c	1	11a	1	12a
2	16a	2	21b	2	15b	2	10b	2	15a
3	11a	3	28b	3	16b	3	9a	3	22a
4	13c	4	20a	4	13c	4	12b	4	15b
5	13b	5	33a	5	19b	5	10a	5	16a
6	11c	6	17a	6	17a	6	8b	6	20a
7	14b	7	24a	7	13b	7	9b	7	18a
8	19a	8	17c	8	16a	8	8a	8	13a
9	12a	9	31a	9	21a	9	12a	9	17a
10	17a	10	26a	10	20b	10	11b	10	12b
11	11b	11	19a	11	20a			11	20b
12	15a	12	31b	12	21b			12	13b
13	20a	13	35a	13	14a	HR 9-1,9-2;		13	14b
14	16a	14	28a	14	13a	8-9 (opt)		14	22b
15	14a	15	30a	15	17b			15	14a
16	17b	16	23a	16	19a	SW 6-6;9-3		16	21b
		17	27a	17	15a			17	19a
		18	32b	18	18a	SZ 3-5		18	23a
ing: * HR 4-4/4-6;		19	21a					19	21a
6-3;11-5		20	25b			AB 30-4,30-5		20	23b
		21	17b	*HR 8-1/8-6				21	19b
SZ 6-6		22	36a						
		23	25a	SW 6-3/6-5					
AB 9-5;9-6		24	18a					* HR 9-3/9-6; 10-1/10-3	
		25	27b	SZ 7-5/7-8					
		26	32a					AB 30-1/30-3	
		27	29a	AB 27-4;28-1/					
		28	34a	28-7					
		29	32a						

*HR Chapter 7

SW 6-1;6-7

SZ 7-1/7-3;
7-9;7-10

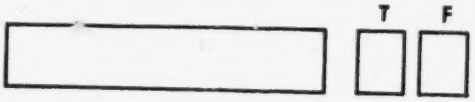
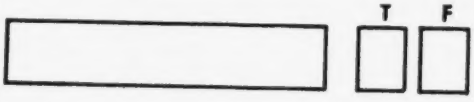
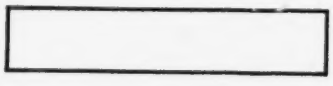
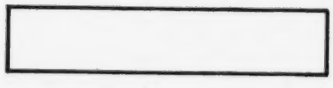
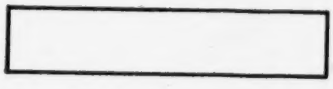
AB 6-1/6-7;
27-1/27-3

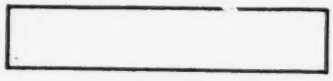
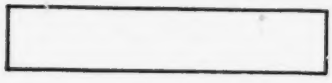
P	STEP	NAME
	0.1	Reading: *HR 4-4/4-6; 6-3; 11-5 SZ 6-6 AB 9-5; 9-6
	0.2	Information Panel, "The Vocabulary of Circular Motion"
1		T F _____ (ans)
	1.1	Information Panel, "The Characteristics of Uniform Circular Motion"
2		A B C D T F
	2.1	If your first choice was correct, advance to 7.1; if not, continue sequence.
3		T F _____ (ans)
4		A B C D
5		A B C D
6		A B C D
7		A B C D T F
	7.1	Audiovisual, CIRCULAR MOTION
8		v_1 = _____ (ans) T F v_2 = _____ (ans)

P	STEP	SECTION SEGMENT 6
	8.1	Information Panel, "Centripetal Force"
9		T F _____ (ans)
	9.1	If correct, advance to P 14; if not, continue sequence.
10		A B C D
11		A B
12		_____ (ans)
13		T F _____ (ans)
14		T F _____ (ans)
15		A B C D T F
16		T F _____ (ans)
	16.1	Homework: HR Problem 4-20.

P	STEP	NAME
	0.1	Reading: *HR Chapter 7 SW 6-1; 6-7 SZ 7-1/7-3; 7-9; 7-10 AB 6-1/6-7; 27-1/27-3
	0.2	Information Panel, "Work Done by a Constant Force"
1		A B C D T F
2		T F (ans)
	2.1	If correct, advance to 4.1; if not, continue sequence.
3		A B C D
4		T F (ans)
	4.1	Audiovisual, WORK WHEN FORCE VARIES IN MAGNITUDE AND DIRECTION
	4.2	Information Panel, "Work Done by a Varying Force"
5		A B C D T F
	5.1	If your first choice was correct, advance to 8.1; if not, continue sequence.
6		(ans)
7		(ans)

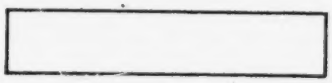
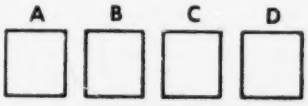
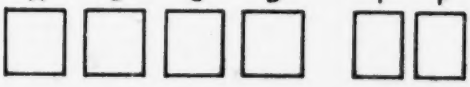
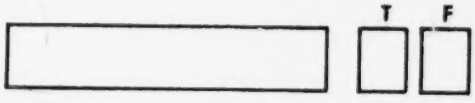
P	STEP	SECTION SEGMENT 7
8		A B C D T F
	8.1	Information Panel, "Algebraic Signs in Work Problems"
9		A B C D T F
	9.1	Information Panel, "Unit Systems for Work and Power"
10		A B C D T F
	10.1	If your first choice was correct, advance to 14.1; if not, continue sequence
11		A B C D
12		(ans)
13		(ans)
14		A B C D T F
	14.1	Information Panel, "Kinetic Energy"
	14.2	Audiovisual, KINETIC ENERGY
15		T F (ans)
	15.1	If correct, advance to 17.1; if not, continue sequence.

P	STEP	NAME
16		
		_____ (ans)
17		 T F
		_____ (ans)
	17.1	Information Panel, "The Work-Energy Theorem"
18		 T F
		_____ (ans)
	18.1	If correct, advance to 23.1; if not, continue sequence.
19		A B C D
20		A B C D
21		A B C D
22		
		_____ (ans)
23		 T F
		_____ (ans)
	23.1	Information Panel, "Composite Problems Involving Work and Energy"
24		 T F
		_____ (ans)
	24.1	If correct, advance to P 27; if not, continue sequence.

P	STEP	SECTION SEGMENT 7
25		
		_____ (ans)
26		 T F
		_____ (ans)
27		A B C D T F
	27.1	If your first choice was correct, advance to 29.1; if not, continue sequence.
28		A B C D
29		A B C D T F
	29.1	Homework: HR Problem 7-8

P	STEP	NAME
	0.1	Reading: *HR 8-1/8-6 SW 6-3/6-5 SZ 7-5/7-8 AB 27-4; 28-1/28-7
	0.2	Information Panel, "Potential Energy and Conservative Forces"
	0.3	Audiovisual, POTENTIAL ENERGY
1		A B C D T F
	1.1	If your first choice was correct, advance to 4.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D T F
	4.1	Information Panel, "Conservation of Energy"
	4.2	Audiovisual, CONSERVATION OF ENERGY
5		A B C D T F
	5.1	If your first choice was correct, advance to 8.1; if not, continue sequence.
6		_____ (ans)

P	STEP	SECTION SEGMENT 8
7		_____ (ans)
8		A B C D T F
	8.1	Information Panel, "Potential Energy When the Resultant Force Varies"
9		T F _____ (ans)
	9.1	If correct, advance to 12.1; if not, continue sequence.
10		A B C D
11		_____ (ans)
12		A B C D T F
	12.1	Information Panel, "Energy in Springs"
13		A B C D T F
	13.1	If your first choice was correct, advance to 17.1; if not, continue sequence.

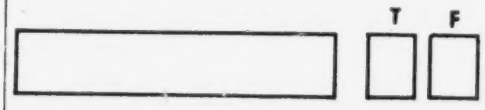
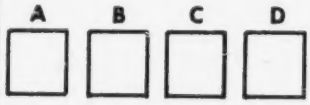
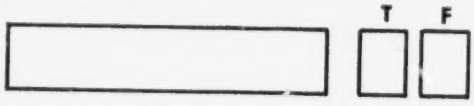
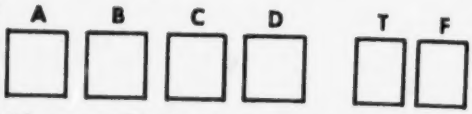
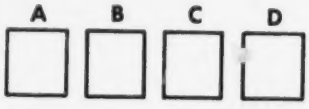
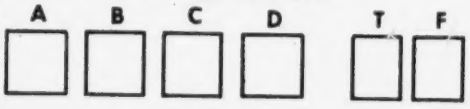
P	STEP	NAME	P	STEP	SECTION	SEGMENT 8
14		(ans)				
15		 (ans)				
16						
17						
	17.1	Information Panel, "A Composite Problem Using Conservation of Energy"				
18		 (ans)				
	18.1	Homework: HR 8-16				

P	STEP	NAME
	0.1	Reading: HR 9-1, 9-2; 8-9 (opt) SW 6-6; 9-3 SZ 3-5 AB 30-4, 30-5
	0.2	Information Panel, "Coordinates of the Center of Mass of a System of Particles"
1		A B C D T F
	1.1	If your first choice was correct, advance to P 4; if not, continue sequence.
2		_____ (ans)
3		A B C D T F
4		A B C D T F
	4.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
5		_____ T F _____ (ans)
	5.1	Information Panel, "Movement of the Center of Mass"
	5.2	Audiovisual, MOVEMENT OF CENTER OF MASS
6		_____ T F _____ (ans)

P	STEP	SECTION SEGMENT 9
	6.1	If your first choice was correct, advance to 10.1; if not, continue sequence.
7		A B C D
8		_____ (ans)
9		A B C D
10		_____ T F _____ (ans)
	10.1	Homework: HR 9-4

P	STEP	NAME
	0.1	Reading: *HR 9-3/9-6; 10-1/10-3 AB 30-1/30-3
	0.2	Information Panel, "The Momentum of a Particle"
1		T F _____ (ans)
	1.1	If correct, advance to 4.1; if not, continue sequence.
2		A B C D
3		_____ (ans)
4		T F _____ (ans)
	4.1	Information Panel, "Momentum of a System of Particles"
5		A B C D T F
	5.1	If your first choice was correct, advance to 9.1; if not, continue sequence.
6		A B C D
7		A B C D
8		A B C D

P	STEP	SECTION SEGMENT 10
9		A B C D T F
	9.1	Information Panel, "The Second Law in Terms of Momentum"
10		T F _____ (ans)
	10.1	If correct, advance to 12.1; if not, continue sequence.
11		A B C D
12		T F _____ (ans)
	12.1	Information Panel, "Conservation of Momentum"
	12.2	Audiovisual, CONSERVATION OF MOMENTUM
13		T F _____ (ans)
	13.1	If correct, advance to P 16; if not, continue sequence.
14		_____ (ans)
15		T F _____ (ans)

P	STEP	NAME	P	STEP	SECTION	SEGMENT 10
16		 _____(ans)				
	16.1	If correct, advance to P 19; if not, continue sequence.				
17						
18		 _____(ans)				
19						
	19.1	If your first choice was correct, advance to 21.1; if not, continue sequence.				
20						
21						
	21.1	Homework: HR 9-15				

INFORMATION PANEL The Vocabulary of Circular Motion

OBJECTIVE

To define and interrelate some of the basic terms used in discussing circular motion.

As soon as you start this segment, you become involved with a few words and phrases that are worth a brief discussion.

REVOLUTIONS PER MINUTE (rev/min) and REVOLUTIONS PER SECOND (rev/sec). A particle completes a single revolution in a circle when it moves from any arbitrary starting point, around the circle, and back to the same point ready to start a second identical circular sweep. If it completes 5 such revolutions in one minute, it is said to have a FREQUENCY of 5 rev/min. If the frequency of a given particle in circular motion is 120 rev/min, it may also be expressed as 2 rev/sec.

TANGENTIAL VELOCITY (v). This is a vector quantity which expresses the instantaneous speed of the particle in a direction tangent to the circle in which it moves. If the particle's speed is constant, then the magnitude of the tangential velocity is constant but its direction is always changing. The tangential velocity vector is always perpendicular to the radius of the circle of rotation.

PERIOD (T). The period is the time required to complete one revolution. If the frequency (f) of the motion is, say, 2 rev/sec, then the period T is 1/2 sec. Clearly, period and frequency are inversely related, one being the reciprocal of the other.

$$T = 1/f \qquad \text{or} \qquad f = 1/T$$

Period may be expressed in any convenient unit of time while frequency is generally expressed in reciprocal time units. That is, the frequency of the particle above is 2 sec^{-1}.

ANGULAR VELOCITY (ω). This quantity is most conveniently expressed in radians per second (rad/sec), and is defined as the angle swept out by the radius vector of the particle per unit time. Although angular velocity is a vector quantity, we will consider only the magnitude of the vector, expressed in rad/sec, at this time. Suppose you were dealing with a particle which has a period of 1 sec. This would mean that the radius sweeps out 2π radians in 1 sec, hence the angular velocity would be 2π radians per second. If the period were half of that, that is, 1/2 sec, the motion would be twice as fast and the

next page

continued

angular velocity would then be 4π radians per second. Thus, period and angular velocity are related as follows:

$$\omega = 2\pi/T$$

and since $T = 1/f$, then:

$$\omega = 2\pi f \ .$$

DISTANCE MOVED BY PARTICLE. The instantaneous velocity of a particle in circular motion is related to the angular velocity by the expression:

$$v = \omega r$$

where r is the radius. If the particle has a constant speed of 25 cm/sec on the circle, then the distance it moves in an interval of time Δt is:

$$d = v\Delta t$$

$$= \omega r \Delta t.$$

The distance moved in 10 sec would be 250 cm. Should the angular velocity be known, the tangential speed v may be readily obtained from the expression immediately above. The radius r must, of course, be given, too.

In the portion of the work that follows, you will be asked to (1) answer descriptive questions dealing with the vocabulary presented above; (2) solve numerical problems involving these terms and phrases.

PROBLEMS

1. The rim of a rotating bicycle wheel has a tangential velocity of 30 m/sec. If 0.5 m is the radius of the rotating wheel, how many revolutions per *minute* (rev/min) would be recorded by a tachometer? (A tachometer is an instrument used to measure revolutions per minute.)

INFORMATION PANEL Characteristics of Uniform Circular Motion

OBJECTIVE

To discuss the significance of radial acceleration in uniform circular motion.

The phrase "uniform circular motion" describes a special case of circular motion in which the particle moves with constant speed, traversing equal lengths of arc in equal times. The magnitude of the tangential velocity is constant but its direction changes from instant to instant.

Students tend to think of acceleration in terms of changing speed, relating it to cars, planes, or ships as they speed up or slow down. However, the definition of acceleration includes changing direction as well as changing speed. A particle in uniform circular motion is accelerating despite the fact that the particle moves with constant speed.

The direction of the acceleration of a particle in uniform circular motion is radially inward and is called *centripetal acceleration:*

$$a_r = v^2/r \quad \text{in} \quad m/sec^2, \ ft/sec^2, \ \text{etc.}$$

where v is the tangential velocity and r is the radius of the circle. Centripetal acceleration, like linear acceleration discussed earlier, is a vector quantity.

You are expected to be able to apply the concept of centripetal acceleration to (1) answer descriptive questions in which it is involved; (2) solve problems in which centripetal acceleration must be considered.

2. A particle moves at constant speed in a circular path of radius r. The particle makes one complete revolution every second. Calculate the acceleration of the particle if r = 0.5 m.

A. 19.8 m/sec
B. 12.6 m/sec^2
C. 10.8 m/sec^2
D. 1.98 m/sec^2

3. When a particle moves in a circular path with constant speed, it is accelerating because the *magnitude* of its velocity is changing.

4. "Uniform circular motion" refers to:

A. any circular motion
B. circular motion with tangential acceleration
C. circular motion without any acceleration
D. circular motion with constant speed

5. Choose the one correct statement below pertaining to a particle in uniform circular motion.

A. Centripetal acceleration is directed radially outward from the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, but not from a change in speed.

B. Centripetal acceleration is directed radially inward toward the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, but not from a change in speed.

C. Centripetal acceleration is directed radially inward toward the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, and from a change in speed.

D. Centripetal acceleration is directed radially outward from the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, and from a change in speed.

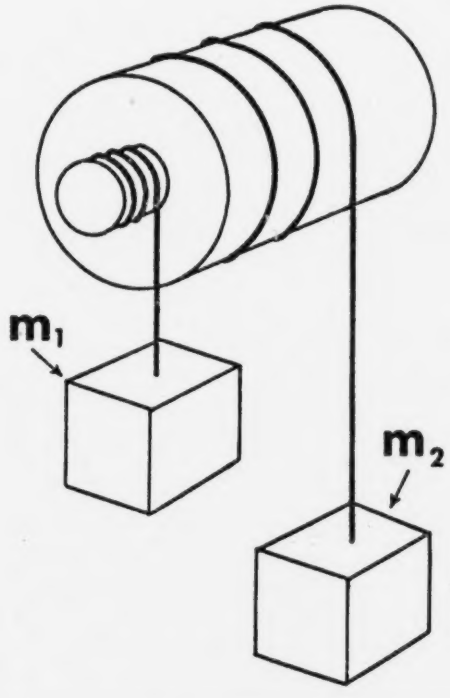
6. The magnitude of acceleration for a particle undergoing uniform circular motion is v^2/r. In this expression, r is the radius of the circle and v refers to

A. the magnitude of the tangential velocity of the particle
B. the magnitude of the outward velocity of the particle (radially outward)
C. the magnitude of the inward velocity of the particle (radially inward)
D. the radial speed in the radial direction of the particle

7. A particle of mass m is moving in a horizontal circle of radius r and making f revolutions per second. If the radius is tripled and the frequency is doubled, find the ratio of centripetal accelerations when the radius is increased from r to 3r.

A. 1/6
B. 1/12
C. 1/3
D. 2/3

8. Two blocks are lowered by a winch made of two concentric cylinders. The smaller cylinder has a radius of 0.04 m, and the larger cylinder has a radius of 1 m. If the winch turns at 3 rev/min, what are the vertical velocities of block one and block two (v_1 and v_2)?



INFORMATION PANEL — Centripetal Force

OBJECTIVE

To apply the concept of centripetal force to problem situations in uniform circular motion.

You may find this line of reasoning helpful in arriving at the correct concept of centripetal force:

*If there is no unbalanced force acting on a moving particle, it will continue to move with constant speed in a straight line;

*A particle in uniform circular motion does not move in a straight line, hence it must undergo acceleration;

*This acceleration is directed radially inward, hence there must be an unbalanced force acting radially inward; we call this force the centripetal force F_c.

*The magnitude of the centripetal force is given by:

$$F_c = mv^2/r$$

When you label a force as "centripetal", you are merely stating that it acts inward toward the center of rotation, but this name gives no information about the nature of the force, nor does it tell anything about the body that is responsible for it. A centripetal force is *not* a new type of force but is so called only because the name is descriptive of its behavior. For a stone whirled in a horizontal circle at the end of a string, the centripetal force is an elastic force provided by the string; for a satellite revolving around the Earth, the centripetal force is gravitational attraction; a charged particle circling within the "dees" of a cyclotron is subjected to a magnetic centripetal force; and so on.

The problems you will encounter in this section are largely of the composite or multistep variety in which centripetal force is but one link in the chain of reasoning. Before starting to work out such problems, you are urged to organize your material in writing as follows:

(1) Make a list of all the "knowns" given in the problem statement in symbolic form. For example, if you are told that the mass is 2 kg, the length 98 cm, and the angle 30° (see P 8), write

Given: $m = 2$ kg
$L = 98$ cm $= 0.98$ m
$\theta = 30°$

(2) Write down the "unknowns", the quantity you must ultimately find, again in symbolic form. In P 8, the unknown is the period τ so

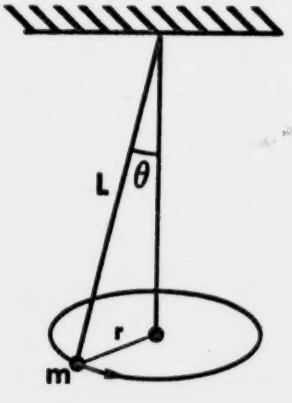
Find: τ (period)

(3) Next, write all the equations that appear to relate these quantities to each other and use them to help you *express all subsidiary unknowns in terms of the knowns.*

(4) You must finally obtain an equation in which only one unknown remains before you can substitute numbers.

In this section, you are expected to (1) analyze the relationships between quantities involved in describing centripetal force; (2) solve problems in which centripetal force is the central concept; (3) solve problems in which centripetal force is a subsidiary consideration, that is, only one of a number of concepts which must be interrelated to solve the problem successfully.

9. The figure shows a mass m = 2 kg revolving in a horizontal circle. The mass is suspended from a string 98 cm in length. The motion of the string traces out a cone. If the string makes an angle of 30° with the vertical, how long does it take for the mass to make one revolution?



10. A body of mass m revolves in a circle of radius r with speed v. What is the magnitude of the centripetal force, F_c, acting on the mass?

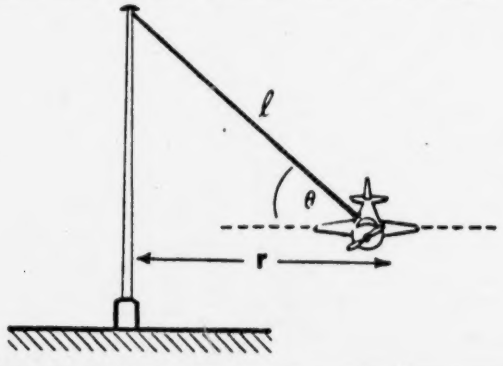
A. $F_c = mv^2/r^2$
B. $F_c = mv^2/r$
C. $F_c = mv^2r^2$
D. $F_c = mv^2r$

11. A man made satellite orbits the Earth in a circular path of radius r. The centripetal force and the force due to gravitational attraction acting on the satellite are

A. the same force
B. different forces

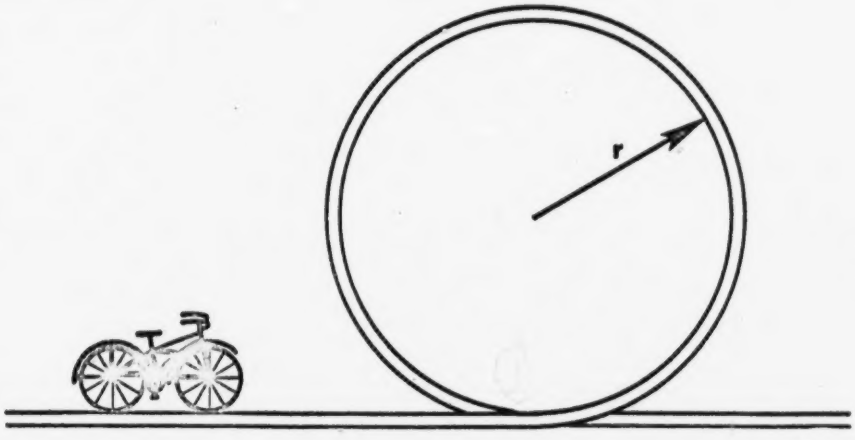
12. A boy spins a 2-kg stone in a horizontal circle at the end of a string. The string is 2 m long, and has a maximum breaking strength of 100 nt. What is the maximum speed the stone may have without breaking the string?

13. At a carnival, cars in the "airplane ride" are suspended from a cable 64 feet long attached to a vertical pole which rotates at 1 radian per second. Find the angle θ that the cable makes with the horizontal.



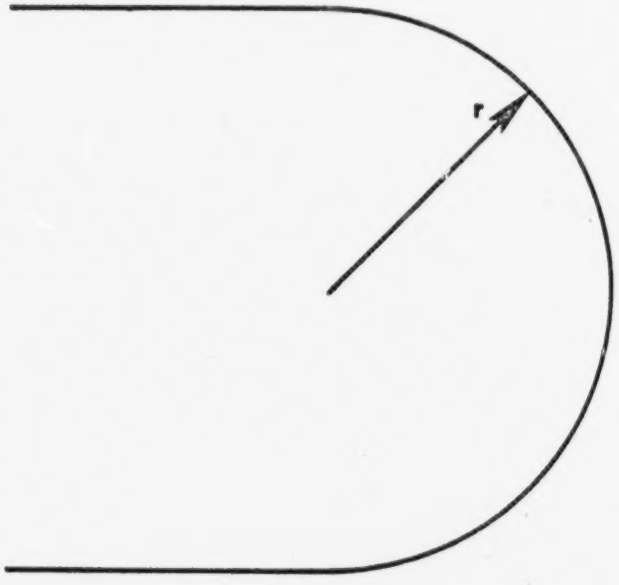
14. A copper penny is placed 4 inches from the center of a hi-fi record. The record plus penny are then placed on a phonograph turntable (33 1/3 rev/min) and the switch is turned on. The coefficient of static and kinetic friction are 0.1 and 0.05 respectively. At what angular velocity will the penny begin to slide?

15. A man plans to perform the loop-the-loop with his bicycle at the county fair (see the diagram below). The radius r is equal to 10 ft. What is the minimum speed at which he can safely perform the stunt?

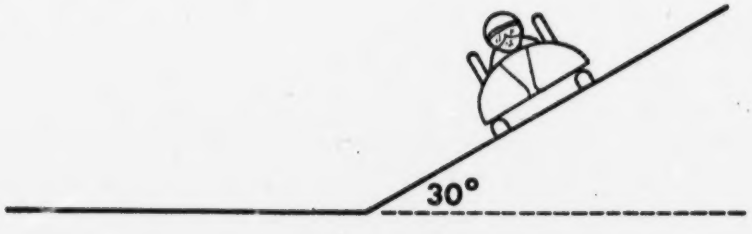


A. depends on the man's mass
B. 12.2 mi/hr
C. 20 ft/sec
D. 9.6 mi/hr

16. A bobsled speeds around the curve shown in the figure below. The curve has been well iced and can be considered frictionless. The sled moves in a circular arc of radius = 100 m and banking angle of 30°; what is its speed?



Curve view from the top



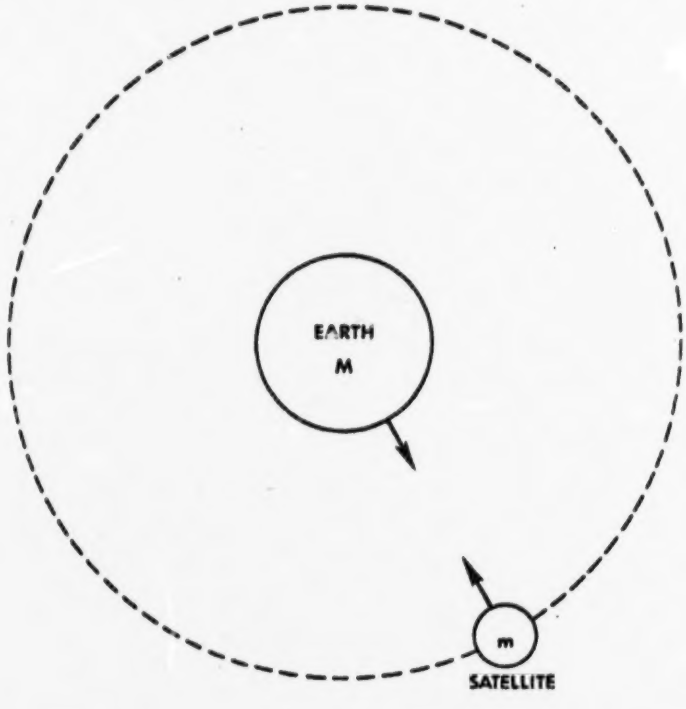
Bobsled on banked curve

[a] CORRECT ANSWER: FALSE (change *magnitude* to *direction*)

Speed has been defined as the magnitude of the velocity vector; in this problem, speed is constant. Since the particle travels in a circle, the *direction* of the velocity is changing at a constant rate. One would anticipate that the particle has a constant acceleration due to its constant change in direction.

[b] CORRECT ANSWER: A

A - As the satellite circles the Earth, only one force pulls the satellite in a circular orbit. This force is an attractive force that exists between any two masses. Since the satellite moves in a circle, this force must equal mv^2/r.



[c] CORRECT ANSWER: A

A - The tangential velocity v is related to the radial acceleration a_r by

$$a_r = v^2/r$$

[a] CORRECT ANSWER: 1.85 sec

Your reasoning may be something like this:

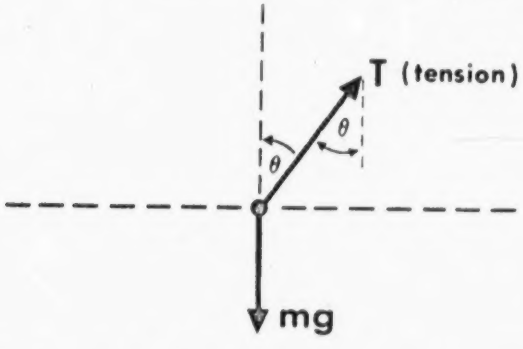
1. In order to find the time for one revolution of the mass, you must know the velocity of the mass and the path length of one revolution.

2. One revolution is $2\pi r$ in length where r is the radius of the horizontal circle in which the mass moves.

3. The magnitude of the tangential velocity v can be found from its relationship to the centripetal force, F_c,

$$F_c = mv^2/r$$

4. In order to determine this force, a free-body diagram is in order.



5. The only force acting in the radial direction is T sin30° and thus this force must be equal to the centripetal force.

Now in equation form: $\Sigma F_y = T\cos30° - mg = 0$ (because $a_y = 0$)

$T\sin30° = mv^2/r$ (because the mass moves in a circle)

Solving for v we obtain

$$v = \sqrt{rg\tan30°}$$

and, of course, from the diagram, $r = L\sin30°$.

Finally, the time for one revolution would be

$$\tau = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{\sqrt{rg\tan30°}} = \frac{2\pi\sqrt{r}}{\sqrt{g\tan30°}} = 2\pi\sqrt{\frac{L}{g}\cos30°} = 1.85 \text{ sec}$$

TRUE OR FALSE? The angle θ in the free-body diagram is 30° for this problem.

[a] CORRECT ANSWER: 573 rev/min

The distance traveled in one revolution is equal to the circumferance, or $2\pi r$, of the circle. This distance divided by the velocity would give us the time for one revolution. The time for one revolution is called the period and is given by the symbol T.

$$T = \frac{2\pi r}{v} = \text{time per revolution}$$

You might also notice that

$$\frac{1}{T} = \text{revolutions/time} = \text{frequency} \equiv f$$

which is the required quantity.

We have

$$f = v/2\pi r$$
$$= 9.55 \text{ revolutions/sec}$$

This can be converted to revolutions per minute as follows:

$$f = 9.55 \frac{\text{rev}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} = 573 \frac{\text{rev}}{\text{min}}$$

TRUE OR FALSE? The period of a rotation increases as the number of revolutions per minute increases.

[b] CORRECT ANSWER: B

B - The word "centripetal" means center-seeking or toward the center. The velocity of a particle in uniform circular motion changes in direction, but the speed (or magnitude) remains constant.

[c] CORRECT ANSWER: D

D - The case of a particle moving in a circle with constant speed is called uniform circular motion. The velocity vector changes continuously in direction but not in magnitude.

[a] CORRECT ANSWER: B

B - The most dangerous part of the stunt will come when the man on the bicycle is upside down at the top of the loop. At that point he is most likely to leave the track. "Just about to leave the track" means that the normal force due to the surface of the track is approaching zero. Therefore, we should solve for this extreme condition to obtain our minimum safe speed.

At the top point, traveling at a velocity v, the only force acting on the man is his weight. Therefore, we obtain

$$\Sigma F_y = -mg = ma$$

This tells us the acceleration is downward, in the radial direction, and equal to the acceleration of gravity. Then

$$a = a_r = v^2/r$$

and

$$v = \sqrt{rg}$$

TRUE OR FALSE? The minimum permissible speed of the bicycle in a loop-the-loop situation is independent of the mass of bicycle and rider.

[b] CORRECT ANSWER: B

B - For a particle moving in a circle of radius r and frequency f, the centripetal acceleration is given by

$$a_r = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r$$

Let a_{1r} be the acceleration when the radius is r and a_{3r} be the acceleration when the radius is 3r. Therefore,

$$\frac{a_{1r}}{a_{3r}} = \frac{4\pi^2 f^2 r}{4\pi^2 (2f)^2 (3r)} = \frac{1}{12}$$

TRUE OR FALSE? In this problem, a_r symbolizes tangential acceleration.

[a] CORRECT ANSWER: 10 m/sec

Some basic facts about circular motion:

1. Circular motion is always accelerated motion. This is due to the fact that even if the speed is constant, there is a constantly changing direction.

2. From Newton's second law we know that acceleration implies the presence of a force. In a uniform circular motion, this force is called centripetal force. If the centripetal force were not present, the object would fly off at a tangent to the circle. Thus, the centripetal force is the force that holds the object in a circular orbit. The magnitude of the forces can be found from the following:

$$F = mv^2/r$$

where

v = tangential velocity (speed)

r = radius, and

m = mass

Most problems involving uniform circular motion require that the total forces which constrain a body to move in a circle are equated to mv^2/r.

In the present problem, when F = 100 nt and r = 2 m, the speed of the stone is

$$v = \sqrt{Fr/m}$$

$$= 10 \text{ m/sec}$$

[a] CORRECT ANSWER: 30 rev/min

The only force available for keeping the penny in a circular path is the force of friction (static). Let's solve for the angular speed at which the static frictional force would be maximum.

$$\Sigma F_y = N - mg = 0$$

$$\Sigma F_x = \mu_s N = ma_r$$

where μ_s is the coefficient of static friction and

$$a_r = \text{radial acceleration} = \omega^2 r$$

Therefore,

$$\mu_s mg = m\omega^2 r$$

Solving for ω, we obtain

$$\omega = \sqrt{\frac{g\mu_s}{r}}$$

Substitution of numerical values yields

$$\omega = \sqrt{\frac{32 \times 0.1}{1/3}}$$

$$\omega = 3.1 \text{ rad/sec}$$

Therefore,

$$\text{rev/min} = 3.1 \frac{\text{rad}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times \frac{1}{2\pi} \frac{\text{rev}}{\text{rad}} = 30 \text{ rev/min}$$

Thus the penny will slide off when the turntable attains the speed of 30 rev/min.

TRUE OR FALSE? The coefficient of kinetic friction must be used in the solution of this problem.

[a] CORRECT ANSWER: B

B - One way to check the answer is to check the dimensions of the answer. Observe that the dimensions of your answer agree with the dimensions ML/T^2 of any force, where M = mass, L = length, and T = time. In our problem,

$$F = mv^2/r,$$

which has dimensions

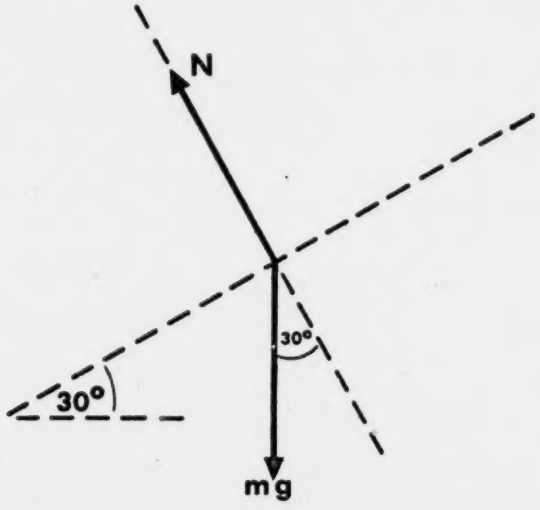
$$ML^2/T^2L$$

reducing to the dimensions of force, ML/T^2.

Remember that this force is always directed towards the center of the circle.

[b] CORRECT ANSWER: 23.8 m/sec

A free-body diagram of the sled in the curve would indicate only two



forces acting on the sled. The normal force must provide a component to balance the weight and a component to cause the radial acceleration.

next page

continued

$$\Sigma F_y = N \cos 30° - mg = 0 \quad \text{(from } a_y = 0\text{)}$$

$$N \sin 30° = mv^2/r \quad \text{(radial force must equal } mv^2/r \text{ to maintain circular motion)}$$

Eliminate N and solve for v:

$$v = \sqrt{rg \tan 30°}$$

Notice that the mass of sled and passengers is not important, so the bob-sled course is properly banked for people of all sizes.

TRUE OR FALSE? The vertical component of the normal force N is not equal in magnitude to the weight mg.

[a] CORRECT ANSWER: C

C - In all problems one should carefully decide on three distinct questions:

(a) What must I find? "Unknowns"

(b) What information has been given? "Knowns"

(c) What equations do I know that relate the "knowns" to the "unkowns"?

In this problem for example,

GIVEN	FIND
1) constant speed	radial acceleration (a_r)
2) circular path	
3) radius of circle	
4) time to travel one revolution	

Since the speed is constant, the particle has only radial acceleration and no tangential component of acceleration. An equation relating some of these quantities is

$$a_r = v^2/r$$

next page

continued

We must determine the velocity, v.

Possible Equation: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

We know that the particle travels one revolution in one second. The distance traveled in one revolution is the circumference of the circle, or $2\pi r$. Therefore,

$$v = \frac{2\pi r}{1} \frac{\text{meters}}{\text{second}}$$

Hence $$a_r = \frac{4\pi^2 r^2}{r} = 4\pi^2 r$$

TRUE OR FALSE? The distance traveled by a particle in making two complete revolutions in circular motion is $4\pi r$ where r is the radius of rotation.

[a] CORRECT ANSWER: $v_1 = 0.01256$ m/sec, $v_2 = 0.314$ m/sec

The ropes unwind at the same speed with which the cylinders turn; that is, their tangential velocities. Tangential velocity v is directly proportional to the radius r. As you might recall,

$$v = \omega r$$

where ω = angular velocity in radians/second.

In our problem,

$$\omega = 3 \frac{\text{rev}}{\text{min}} \times \frac{1}{60} \frac{\text{min}}{\text{sec}} \times 2\pi \frac{\text{rad}}{\text{rev}} = 0.314 \frac{\text{rad}}{\text{sec}}$$

and $$v_1 = \omega r_1$$

becomes $$v_1 = 0.314 \frac{\text{rad}}{\text{sec}} \times 0.04 \text{ m} = 0.01256 \text{ m/sec}$$

Similarly, $$v_2 = \omega r_2$$

and gives $$v_2 = 0.314 \frac{\text{rad}}{\text{sec}} \times 1.0 \text{ m} = 0.314 \text{ m/sec}$$

TRUE OR FALSE? In this problem, the angular velocity of the smaller cylinder is exactly the same as the angular velocity of the larger cylinder.

[a] CORRECT ANSWER: 30°

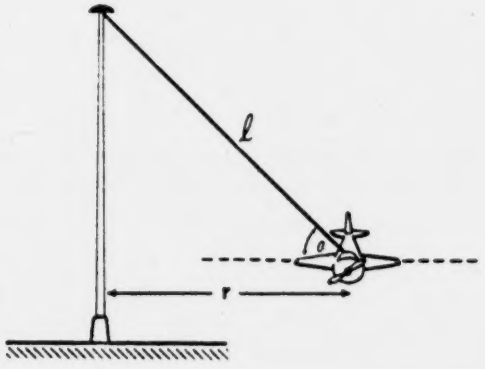
Using Newton's second law, the sum of the horizontal forces, ΣF_h, is

$$\Sigma F_h = T\cos\theta = ma \qquad (1)$$

Since the particle is moving in a circular path with constant speed, the acceleration a is the centripetal acceleration. Therefore, equation (1) may be written as

$$T\cos\theta = m\omega^2 r$$

where r is the radius of the circular path.



From the diagram

$$r = \ell\cos\theta$$

Substitution of $r = \ell\cos\theta$ in equation (1) yields

$$T\cos\theta = m4\pi^2 f^2 \ell\cos\theta \qquad (2)$$

The sum of the forces in the vertical direction, ΣF_v, is

$$\Sigma F_v = T\sin\theta - mg = 0$$

or

$$T\sin\theta = mg \qquad (3)$$

Dividing equation (3) by equation (2) we obtain

$$\sin\theta = \frac{g}{\omega^2\ell}$$

$$= 1/2$$

Therefore,

$$\theta = 30°$$

TRUE OR FALSE? The centripetal force acting on the carnival car is the horizontal component of the tension in the string.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

Work Done by a Constant Force

OBJECTIVE

To calculate the work done by a constant force, that is, a force which varies neither in magnitude nor direction.

In the simplest situation, where the force applied to a body is constant in both direction and magnitude and where the resulting motion occurs in a straight line, we define work as the product of the magnitude of the force and the displacement of the particle on which the force acts.

Since force and displacement are both vectors, care must be taken to use a consistent system of symbols. In our work we will continue to use $\vec{r}$ for the position vector. Displacement will be designated by $\vec{s}$ so that a particle moving from position $\vec{r}_1$ to $\vec{r}_2$ will undergo a displacement of $\Delta\vec{r} \equiv \vec{s} = \vec{r}_2 - \vec{r}_1$. Thus, with this convention, $d\vec{s} \equiv d\vec{r}$ and the two differentials may be used interchangeably although $d\vec{s}$ will be the preferred form.

The work W done by a constant force $\vec{F}$ acting on a body which moves through a displacement $\vec{s}$ is $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$ in which θ is the angle between the two vectors.

If we designate the component of the force in the s-direction as F_s, then

$$F_s = F\cos\theta$$

and so

$$W = F_s s$$

In working through the problems dealing with the work done by a constant force, you will be expected to

(a) justify the conclusion that the work done by centripetal force on a particle moving uniformly in a circle is zero;

(b) calculate the work done on a given mass when moved up an incline by a given distance;

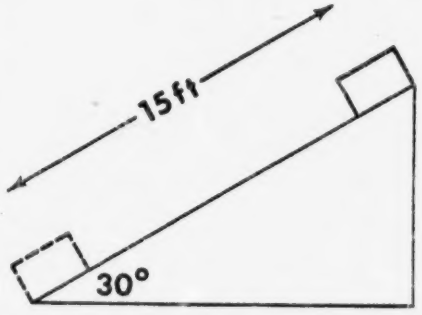
(c) find the work done on a given mass when lifted vertically over a given distance.

PROBLEMS

1. A 2-kg particle is moving in a circle with an angular velocity of 10 rad/sec. The diameter of the circle is 1 m. How much work is done on the particle by the centripetal force during one revolution?

A. 400π j
B. 200π j
C. 100π j
D. Zero j

2.

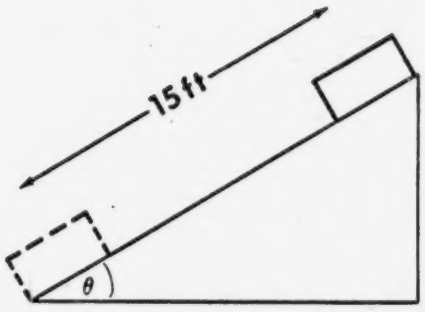


A safe having a mass of 2 slugs is moved up a 30° frictionless inclined plane for a distance of 15 ft. Calculate the work done on the safe.

3. A book of mass m is lifted a vertical distance y near the surface of the Earth. Which of the following expressions gives the work done on the book?

A. (1/2)my
B. my
C. (1/2)mgy
D. mgy

4. A 3-kg block is pushed 15 ft along a frictionless inclined plane. The work expended is 720 ft-lb. Find the angle of incline θ.



INFORMATION PANEL

Work Done by a Varying Force

OBJECTIVE

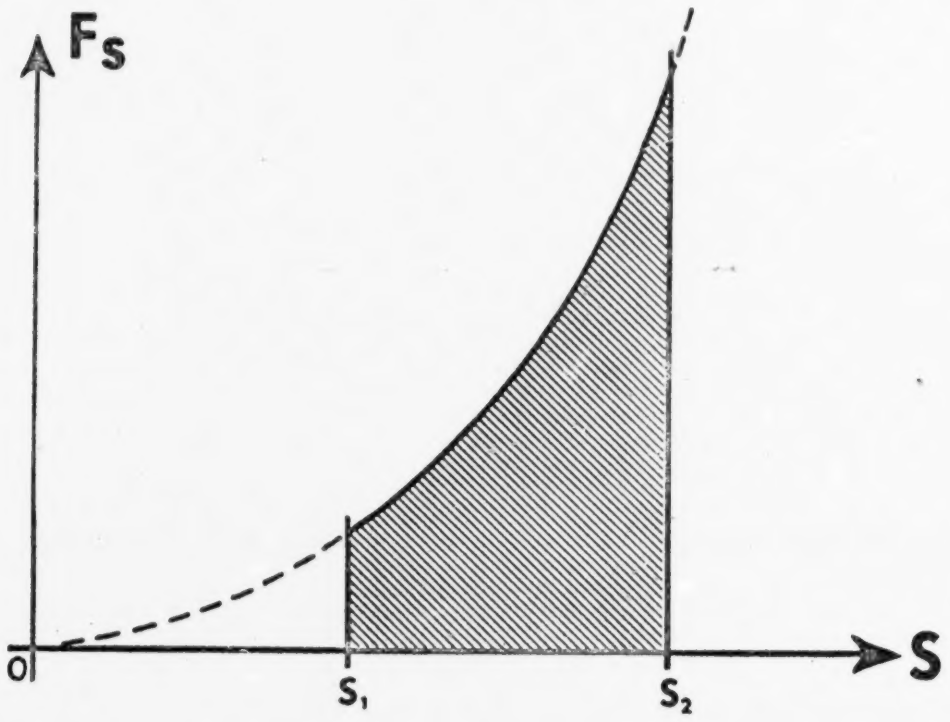
To calculate the work done by a force that varies in a known manner.

The force that causes the displacement of a particle often changes in magnitude and/or direction from instant to instant in a mathematically predictable way. For example, the magnitude of a force is often a function of the displacement of the particle to which the force is applied: as a spring is stretched or compressed, the force needed to produce each successive increment of displacement becomes larger as the end of the spring moves farther from its rest position. In this case, the work done is no longer a simple product of force and displacement.

When the force varies in magnitude, or when the force and displacement are not in the same direction, the work done is given by

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} = \int_{s_1}^{s_2} F_s ds$$

In the graph of F_s vs s, it can be seen that the work due to a displacement from s_1 to s_2 is the area under the curve between the end points s_1 and s_2.



You will find the problems in this section require that you be able to

(a) calculate work when the force is given as a function of displacement (in equation form);

(b) calculate work when the dependence of force on displacement is shown in the form of a graph;

(c) calculate work in the special cases when the force is a simple restoring force, or a gravitational force near the Earth's surface.

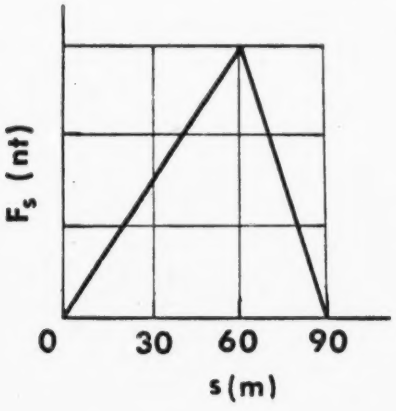
5. A mass m = 2 kg moves in the direction of an applied force varying with displacement according to the equation

$$F = m(\alpha + \beta x^2)$$

where α = 5 m/sec^2, β = 15 m^{-1}sec^{-2}, and x is the displacement. Find the work done on the mass during the first 2 m of its journey.

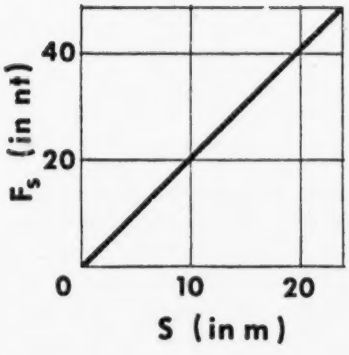
A. 260 j
B. 130 j
C. 100 j
D. 20 j

6.



The graph shows the dependence of F_s on the displacement s. It is found that the work done from s_1 = 0 m to s_2 = 90 m is equal to 450 j. Find the maximum value of F_s.

7.



The diagram shows the linear dependence of F_s on the displacement s. Compute the amount of work done between s_1 = 10 m and s_2 = 20 m.

8. A mass m = 2 kg moves in the direction of an applied force varying with displacement x according to the equation

$$F = m(ax + b)^2$$

where a = 1.0 $m^{-1/2}sec^{-1}$, and b = 2.0 $m^{1/2}sec^{-1}$. Find the work done on the mass during the first 3.0 m of its journey.

A. 130 j
B. 110 j
C. 94 j
D. 78 j

INFORMATION PANEL — Algebraic Signs in Work Problems

OBJECTIVE

To recognize that work may be designated as either positive or negative and to use the accepted sign convention in solving problems that call for it.

It is convenient to assign an algebraic sign to work under certain conditions. Fundamentally, the sign indicates which "body" actually does the work.

Suppose we are trying to find the work done by a body A *on* body B and it happens that body B is actually doing the work on body A. Then as far as our original assumption is concerned, that is, that body A is doing the work on body B, we would have to say that the work W_{AB} is a negative quantity.

By convention, the work done by a physical system on its environment is taken as positive. If the work comes out negative, then we know that the environment had done work on the system rather than the other way around. In problem 9 that follows, the system consists of a spring resting on a frictionless table with one end of the spring fixed in position; the "environment" in this problem is a 5-kg mass. As you work through the problem, the need for the sign convention will become apparent.

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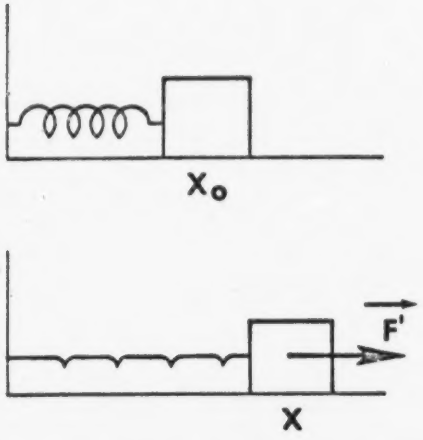


A spring is said to obey Hooke's law if the force necessary to stretch or compress the spring is directly proportional to the amount of stretching or compression. Within limits, most springs obey Hooke's law.

If the equilibrium position of the free end of a spring is x_o, the force $\vec{F}'$ necessary to stretch the spring to a new position x is given by

$$\vec{F}' = k(x - x_o)$$

where k is the force constant of the spring. In accord with our convention, the work done by the spring on its environment is positive.

From Newton's third law, the force of the spring on the block is

$$\vec{F} = -\vec{F}' = -k(x - x_o)$$

or, written in scalar form,

$$F = -k(x - x_o)$$

In working through the following problems, it is anticipated that you will be capable of

(a) recognizing that the magnitude of the force at any instant depends on the displacement of the body;

(b) computing the work done on the system *or* by the system, depending on the algebraic sign of the final answer.

9. A 5-kg block is attached to the spring shown in the diagram at the top of the page. The spring, when unstretched, has a length of 0.15 m (including the block), and its force constant k is equal to 2000 nt/m. Compute the work done in moving the block from x_1 = 0.10 m to x_2 = 0.25 m.

A. -750 j
B. 52.5 j
C. -7.5 j
D. -52.5 j

INFORMATION PANEL — Unit Systems for Work and Power

OBJECTIVE

To solve problems in work and power using consistent units throughout the solutions.

For your convenience, we have listed the power and work units commonly used in the three systems with which we deal in this course.

	WORK (energy)	POWER
MKS	joule (j) or newton-meter (nt-m)	watt (w) or joule per second (j/sec)
British	foot-pound (ft-lb)	foot-pound per second (ft-lb/sec) foot-pound per minute (ft-lb/min)
CGS	erg or dyne-centimeter	erg per second (erg/sec)
Misc.	British thermal unit (Btu) kilowatt-hour (kw-hr)	horsepower (hp)

Useful Conversions

1 watt-sec = 1 joule = 1 nt-m
1 erg = 1 dyne-cm = 10^{-7} j
1 hp = 550 ft-lb/sec = 33,000 ft-lb/min = 746 w
1 kw-hr = 3.6×10^{6} j
1 Btu = 778 ft-lb

The questions and problems in this section require that you be able to

(a) use the relationship $P = \vec{F} \cdot \vec{v}$ in numerical and symbolic work;

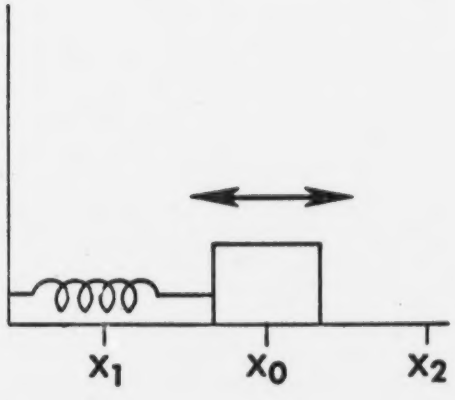
(b) find the instantaneous power supplied by an oscillating spring at a given displacement;

(c) combine work, power and friction concepts in numerical applications.

10. An escalator, inclined at 37° from the horizontal, has a motor that can deliver a maximum power of 10 hp. If the escalator is moving with a constant speed of 2 ft/sec, what is the maximum number of passengers, with an average weight of 150 lb, that the escalator can handle? (Neglect frictional losses.)

A. 30
B. 18
C. 41
D. 31

11.



A mass m attached to a spring with force constant k is oscillating back and forth between points x_1 and x_2 on a frictionless horizontal table. What is the instantaneous power supplied by the spring at the instant the mass passes through the midpoint x_o? (In the following, v is the speed of the mass at the moment it passes through point x_o.)

A. kx_ov
B. $k(x_2 - x_o)v$
C. $k(x_2 - x_1)v$
D. zero

12. A constant horizontal force of magnitude F is required in order to slide a 100-kg block on a horizontal floor with a constant speed of 5 m/sec. The coefficient of kinetic friction between the block and the floor is 0.2. How much power must be supplied by the agency responsible for the force F?

13. A 2500-lb car maintains a constant speed of 60 mi/hr up a road inclined at 20° from the horizontal. Assume that frictional forces can be neglected. How much power does the engine of the car develop?

14. An escalator, inclined at 37° from the horizontal, has a motor that can deliver a maximum power of 5 hp. If the escalator is moving with a constant speed of 1 ft/sec, what is the maximum number of passengers, with an average weight of 150 lb, that the escalator can handle? (Neglect frictional losses.)

A. 18
B. 30
C. 31
D. 41

INFORMATION PANEL — Kinetic Energy

OBJECTIVE

To apply the definition of kinetic energy to descriptive questions; to solve numerical problems involving kinetic energy only.

The kinetic energy of a moving mass is defined as its ability to do work by virtue of its motion. Energy of any kind is a scalar quantity; kinetic energy K is determined for any given body by the relationship

$$K = 1/2\ mv^2$$

in which m = the mass of the body and v = the magnitude of its velocity, or its speed.

next page

continued

A mass moving with constant speed has a constant kinetic energy. The kinetic energy of a mass moving with changing speed varies from instant to instant, hence the instantaneous kinetic energy is a function of the instantaneous speed at the moment in question.

As a check on your understanding of kinetic energy, you must be able to

(a) solve a problem involving the kinetic energy of a projectile at the highest point of its trajectory;

(b) determine the kinetic energy of a given moving mass, giving attention to units, their conversion, and their consistency.

15. A 2-kg particle is projected from ground level with an initial velocity of 20 m/sec, at 60° above the horizontal. Find the kinetic energy of the particle when it reaches its highest altitude; i.e., where the vertical component of the velocity is zero. (Neglect air resistance.)

16. A 3000-lb car is moving with a speed of 60 mi/hr. Find its kinetic energy.

17. The kinetic energy of a 2-kg projectile at the highest point in its trajectory is 25 joules. It is known that the projectile was fired at an angle of 60° from the horizontal. Find the initial speed of the projectile.

INFORMATION PANEL The Work-Energy Theorem

OBJECTIVE

To define the work-energy theorem; to use the theorem for solving symbolic and numerical problems.

The work-energy theorem for a particle states that the work done by the *resultant* force applied to a particle is equal to the change in the kinetic energy of the particle. Or

$$W_R = K - K_o = \Delta K$$

wherein W_R = the work due to the resultant force; K = the kinetic energy of the particle after the work has been done; K_o = the initial kinetic energy before application of the resultant force.

The following can be deduced from the work-energy theorem:

1. For a particle moving with constant speed, there is no change of kinetic energy, hence the work done by the resultant force is zero.

2. The speed of the particle along a given line of motion can be changed only when the resultant force has a component along that line.

3. If the kinetic energy of a particle diminishes, the work done by the resultant force is *negative*; if the kinetic energy increases, the work is *positive*.

4. The kinetic energy of a mass in motion equals the work it can do before it is brought to rest.

5. The units used for work and kinetic energy are identical.

The following problems call for the ability to

(a) solve a problem in kinematics by means of the work-energy theorem;

(b) solve a problem involving a projectile fired at an angle to the horizontal in which you are to determine its kinetic energy upon returning to earth;

(c) determine the work required to double the speed of a given particle;

(d) use the work-energy theorem in calculating the force acting on a body, given the factors needed to find its kinetic energy before and after the force has acted.

18. A block is projected with an initial speed of 8 m/sec, down a frictionless plane inclined 45° from the horizontal. Find the speed of the block after it has traveled for a distance of 2.6 m along the incline. (Use the work-energy theorem in your solution.)

19. A constant force $\vec{F}$ is used to accelerate a body of mass m to a velocity $\vec{v}$. The force is then removed. How much work is done on the object?

A. Fv
B. $mv^2/2$
C. $(F^2/2m)t^2$
D. Fvt

20. A particle of mass m is moving with a speed v. How much work must be done on the particle in order to double its speed?

A. $(1/2)mv^2$
B. mv^2
C. $(3/2)mv^2$
D. $2mv^2$

21. A 2-kg sphere is projected with an initial velocity of 10 m/sec, at 39.3° above the horizontal, from the ground level of a horizontal field. What is the sphere's kinetic energy at the instant it hits the ground? (Neglect air resistance.)

A. 296 j
B. 256 j
C. 196 j
D. 100 j

22. A bullet having a weight of 1 oz, moving with a speed of 600 mi/hr, penetrates a tree trunk to a depth of 11 inches before coming to rest. Calculate the average force exerted on the bullet.

23. A block is projected upward on a frictionless inclined plane with an initial speed of 10 m/sec. The plane is inclined at 45° from the horizontal. Find the speed of the block after it has traveled a distance of 2.6 m along the incline.

INFORMATION PANEL Composite Problems Involving Work and Energy

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OBJECTIVE

To combine the concepts of work and energy with other kinematic and dynamic aspects of physics in solving composite problems.

You have been made aware of the necessity for organizing your work before you begin to substitute numbers in equations dealing with composite problems. All the rules previously described--itemization of the knowns and unknowns, and writing down the interrelating equations--apply equally well to this section of your work.

The remaining problems in this segment are such composite problems. In working these out, you are expected to use the work-energy theorem in solving problems which require that you

(a) first use Newton's second law to calculate an unknown force and then apply the theorem;

(b) make use of the concepts of friction on an inclined plane to find the speed of a block on the plane after it has traveled a specified distance under the action of a constant force;

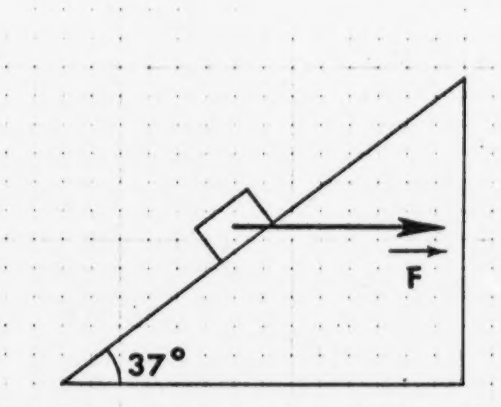
(c) calculate the speed acquired by a mass as the result of the decompression of a spring.

24. A 30-gm bullet, fired with a speed of 300 m/sec, passes through a telephone pole 30 cm in diameter at a point 2 m above ground. The bullet's path through the pole is horizontal and along a diameter. While in the pole the bullet experiences an average force of 2500 nt. If air resistance is neglected, at what horizontal distance from the pole will the bullet hit the ground?

25. A block weighing 16 lb is initially at rest. It is made to move through a distance of 100 ft in 10 sec by a constant force. Find how much work must have been done.

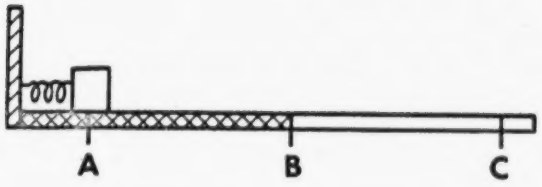
26. A 100-gm rubber ball is held at the bottom of a barrel full of water and then released. While rising to the surface, the ball experiences an average upward force of 490 dynes (this *includes* the weight of the ball). The barrel is 1000. cm high. Find the maximum distance through which the ball will rise above the barrel.

27. A constant horizontal force $\vec{F}$, of magnitude 120 nt, is used to move a 10-kg block up a plane inclined at 37° from the horizontal. If the block starts from rest, and the coefficient of kinetic friction between the block and the plane is 0.200, what is the speed of the block after it has traveled 10 m along the plane?



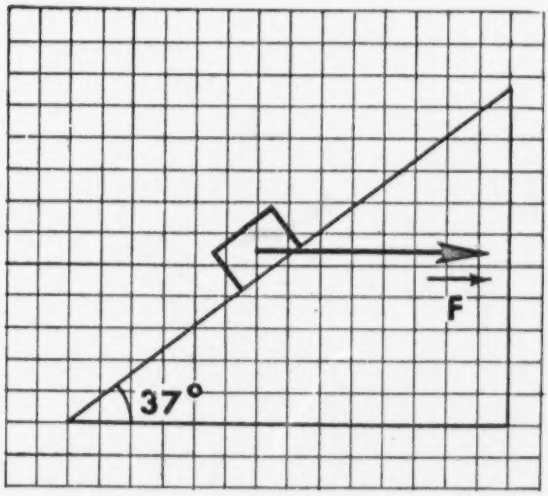
A. 6.56 m/sec
B. 9.55 m/sec
C. 12.8 m/sec
D. 3.76 m/sec

28. A 2-kg block is held at point A on a horizontal floor, with a compressed spring placed between the block and a wall but not attached to the block. The floor surface to the right of B is frictionless. The coefficient of kinetic friction between the surface to the left of B and the block is 0.25. The spring has a force constant of 573 nt/m, and it is compressed to a length 20 cm shorter than its normal length. The distance from A to C is 1 m, and B is midway between A and C. If the block is suddenly released, what will be its speed when it goes past point C?



A. 2.56 m/sec
B. 3.00 m/sec
C. 5.12 m/sec
D. 3.16 m/sec

29. A constant horizontal force $\vec{F}$, of magnitude 120 nt, is used to move a 10-kg block up a plane inclined at 37° from the horizontal. If the block starts from rest, and the coefficient of kinetic friction between the block and the plane is 0.200, how far has the block traveled along the plane when its speed is 2.1 m/sec?



A. 3.1 m
B. 4.1 m
C. 5.1 m
D. 6.1 m

[a] CORRECT ANSWER: 10 nt

The work done is equal to the area of the triangle in the figure. The area of a triangle is given by (1/2)(base) × (height).

Thus, $W = \frac{1}{2}\,bh$, and $h = 2W/b$.

But the height here is the desired F_s maximum. Hence,

$$F_s\ (\max) = \frac{2W}{b} = \frac{2 \times 450}{90} = 10 \text{ nt}$$

[b] CORRECT ANSWER: D

D - The only force acting on the particle is that of gravity, directed along the negative y-axis. Since, however, the net *vertical displacement of the sphere, at the point it hits the ground, is zero,* the net work done by the force of gravity is zero and, therefore, $\Delta K = 0$. Thus,

$$K_f = K_i = (1/2)mv_o^2 = (1/2) \times 2 \times (10)^2 = 100 \text{ j}$$

[c] CORRECT ANSWER: D

D - Given that

$$F = m(ax + b)^2$$

the work done is expressed by

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{s} = m\int_{x_1}^{x_2} (ax + b)^2\, dx$$

$$= m\int_{x_1}^{x_2} (a^2x^2 + 2abx + b^2)\, dx$$

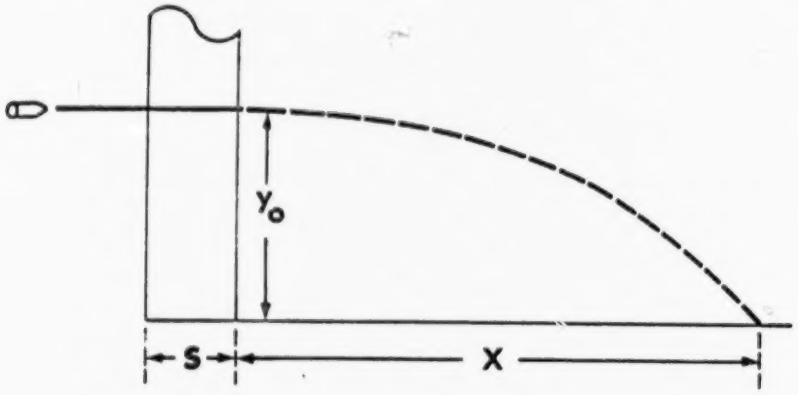
$$= m\left(a^2\,\frac{x^3}{3} + 2ab\,\frac{x^2}{2} + b^2x\right)\Bigg|_{x_1}^{x_2}$$

Set $x_1 = 0$ and $x_2 = 3$, to obtain

$$W = 78 \text{ j}$$

TRUE OR FALSE? The solution of this problem assumes the applied force to be constant over the entire interval from $x_1 = 0$ to $x_2 = 3$.

[a] CORRECT ANSWER: 128 m



We use the work-energy theorem to find the speed of the bullet when it comes out of the pole. (Note that the force is opposite to the direction of motion.)

$$-Fs = \Delta K = \frac{1}{2} mv^2 - \frac{1}{2} mv_o^2$$

or

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 - Fs = \frac{1}{2} (mv_o^2 - 2Fs)$$

and

$$v = \sqrt{\frac{(mv_o^2 - 2Fs)}{m}} \qquad (1)$$

The bullet comes out of the pole horizontally, so $v_{oy} = 0$. Also the final $y = 0$, so from

$$y = y_o + v_{oy}t - \frac{1}{2} gt^2$$

we find the time it takes the bullet to hit the ground

$$t = \sqrt{\frac{2y_o}{g}} \qquad (2)$$

Finally, since there is no acceleration in the x-direction,

$$x = v_{ox}t \qquad (3)$$

next page

continued

In equation (3), v_{ox} is the speed found in (1) and t is the time found in (2); thus,

$$x = \sqrt{\frac{(mv_o^2 - 2Fs)}{m} \frac{2y_o}{g}} \qquad (4)$$

Substituting the given data in (4) we find

$$x = \sqrt{\frac{[30 \times 10^{-3} \times (300)^2 - 2 \times 2500 \times 0.30]\ (2 \times 2)}{(30 \times 10^{-3})\ 9.8}}$$

$$= \sqrt{\frac{(2700 - 1500)\ 4}{0.294}} = 128 \text{ m}$$

TRUE OR FALSE? After passing through the pole, the trajectory of the bullet is parabolic.

[a] CORRECT ANSWER: D

D - The work done by a force $\vec{F}$ in moving a body through a displacement $d\vec{s}$ has been defined as

$$dW = \vec{F} \cdot d\vec{s} \qquad (1)$$

Dividing both sides of (1) by the time differential dt we find

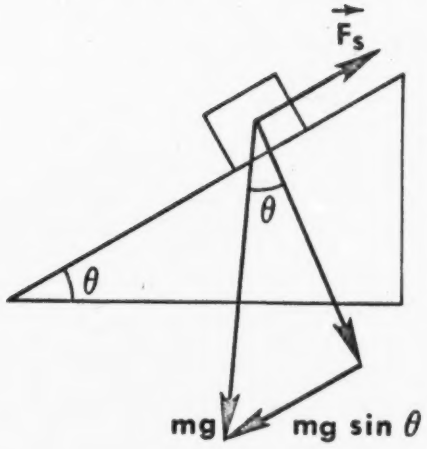
$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v} \qquad (2)$$

But dW/dt is the rate at which work is done by the force; i.e., the power delivered by the force. Hence,

$$P = \vec{F} \cdot \vec{v} \qquad (3)$$

In the present problem we want to find the power when the mass is at the equilibrium point. From the relation $F = -k(x - x_o)$ we see that at $x = x_o$, $F(x_o) = 0$. Therefore, the spring delivers no power at the instant the mass m passes through the mid-point x_o.

[a] CORRECT ANSWER: 30°



The component of the applied force along the inclined plane, F_s, must at least equal the component of the weight along the incline. We have

$$F_s = mg \sin\theta$$

and

$$W = \vec{F}\cdot\vec{s} = F_s s$$

$$= (mg \sin\theta)\, s$$

so

$$\sin\theta = W/mgs$$

$$\theta = \sin^{-1}(W/mgs)$$

$$= \sin^{-1}[720/(3 \times 32 \times 15)]$$

$$= 30°$$

TRUE OR FALSE? The work required to move the block up the plane varies directly with the mass of the block.

[a] CORRECT ANSWER: B

B - A constant force produces a constant acceleration. Using the equation involving a, v, and s, with $v_o = 0$, we get

$$v^2 = 2as$$

or

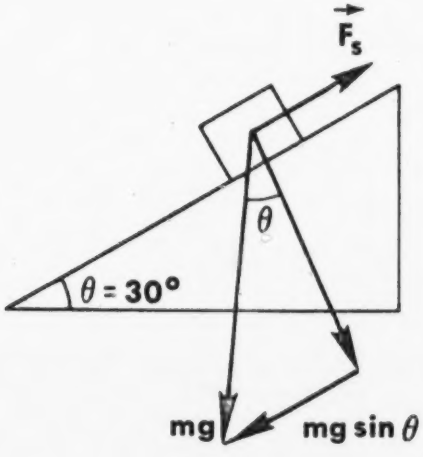
$$a = \frac{v^2}{2s}$$

The force is given by $F = ma$, thus $F = mv^2/2s$. The work done by this force is therefore

$$W = \vec{F}\cdot\vec{s} = Fs\cos 0° = \frac{mv^2}{2s}\, s = \frac{mv^2}{2}$$

[b] CORRECT ANSWER: 480 ft-lb

The component of the applied force along the inclined plane, F_s, must be equal and opposite to the component of the weight along the incline. Thus,



$$F_s = mg\sin\theta$$

and

$$W = \vec{F}\cdot\vec{s} = F_s s = (mg\sin\theta)\, s$$

$$= 2 \text{ slugs} \times 32 \frac{\text{ft}}{\text{sec}^2} \times \frac{1}{2} \times 15 \text{ ft}$$

$$= 480 \text{ ft-lb}$$

(We have used the fact that

$$1 \text{ slug} = 1 \frac{\text{lb-sec}^2}{\text{ft}}.)$$

TRUE OR FALSE? The work done in moving the safe a distance of 15 ft along the plane is independent of the angle of the plane.

[a] CORRECT ANSWER: A

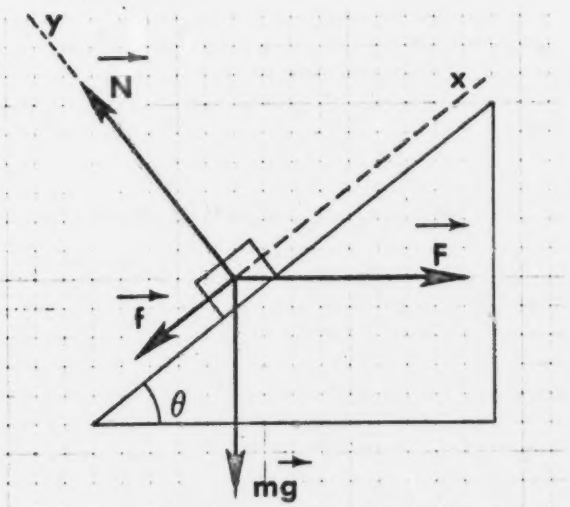


Figure 1

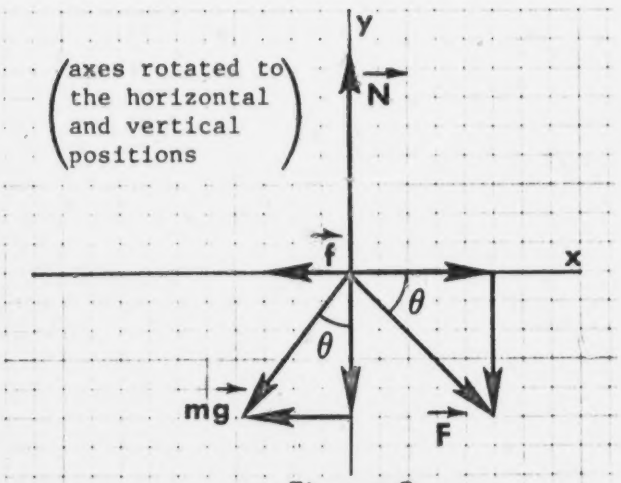


Figure 2

In Figure 1 we show all the forces acting on the block. along with the coordinate system chose for this problem. The forces involved are:

i) the applied horizontal force $\vec{F}$,

ii) the block's weight $m\vec{g}$,

iii) the plane's reaction force $\vec{N}$, and

iv) the force of friction $\vec{f}$ of magnitude $f = \mu N$.

Since the displacement is along the positive x-axis, only the x-components of these forces will contribute to the work done on the block. Therefore, the normal reaction force $\vec{N}$ will not contribute directly to the work done, being perpendicular to $\vec{s}$. It does contribute indirectly, however, since the magnitude of the frictional force $\vec{f}$ is proportional to $\vec{N}$. The direction of $\vec{f}$ is, of course, always opposite to the motion.

The force $\vec{N}$ is

$$\vec{N} = -\Sigma \vec{F}_y$$

or written in scalar form,

$$N = -(-F \sin\theta - mg \cos\theta) = F \sin\theta + mg \cos\theta$$

next page

continued

So the magnitude of the force of friction becomes

$$f = \mu N = \mu(F \sin\theta + mg \cos\theta) \qquad (1)$$

The total work done on the block is (see Figure 2)

$$W = \vec{F}\cdot\vec{s} = s\Sigma F_x = s[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \qquad (2)$$

Using the work-energy theorem $\Delta K = W$, we find

$$\frac{1}{2} mv^2 = W$$

or

$$s = \frac{1}{2} mv^2/[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \qquad (3)$$

$$s = \frac{\frac{1}{2}(10)(2.1)^2}{(120)(0.8) - (10)(9.8)(0.6) - (0.2)[(120)(0.6) + (10)(9.8)(0.8)]}$$

$$= 3.1 \text{ m}$$

TRUE OR FALSE? In this problem, the block comes to rest after traveling 3.0 m up the plane.

[a] CORRECT ANSWER: 3.6×10^5 ft-lb

In using the expression for the kinetic energy

$$K = \frac{1}{2} mv^2$$

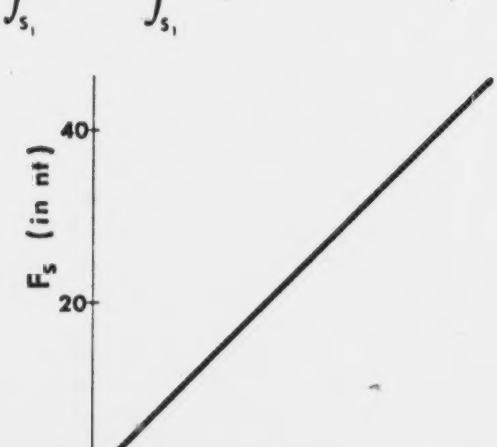
we must make sure that consistent units are used for all the quantities involved. Thus, m = 3000/32 slugs and v = 60 mi/hr = 88 ft/sec, so

$$K = \frac{1}{2}\left(\frac{3000}{32}\right)(88)^2 = 3.63 \times 10^5 \text{ ft-lb}$$

[a] CORRECT ANSWER: 300 j

The work done by a variable force is given by

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} = \int_{s_1}^{s_2} F_s ds$$



From the graph we see that F_s depends linearly on s ($F_s = ks$), with the slope k equal to (40 nt)/(20 m) = 2 nt/m. Hence,

$$W = k\int_{s_1}^{s_2} sds = (1/2)ks^2 \Big|_{s_1}^{s_2} = (1/2)k\ (s_2^2 - s_1^2)$$

$$= (1/2)(2)(400 - 100) = 300\ j$$

The work can also be computed "geometrically." The integral $\int F_s ds$ is equal to the area under the F_s versus s curve, between the specified s-limits. A look at the graph will convince you that the area is indeed 300 nt-m = 300 j.

[a] CORRECT ANSWER: 8 m/sec

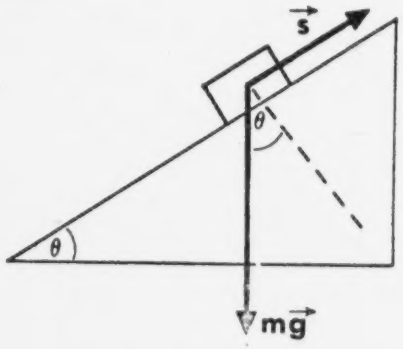
The work done by the block is equal to the change in kinetic energy K of the block,

$$W = \Delta K$$

or

$$W = K_f - K_i$$

All the work on the block is done by the component of weight projected along the incline. From the diagram, we have



$$W = m\vec{g}\cdot\vec{s}$$

$$= mgs\cos(\theta + 90°)$$

$$= -mgs\sin\theta$$

The work-energy theorem becomes

$$-mgs\sin\theta = \frac{1}{2}m(v_f^2 - v_i^2)$$

or

$$v_f^2 = v_i^2 - 2gs\sin\theta$$

Using the given data, v_i = 10 m/sec, s = 2.6 m, and θ = 45°, we find v_f = 8 m/sec.

TRUE OR FALSE? This block will come to rest at the end of its upward journey after traveling a distance of 2.5 m up the plane.

[b] CORRECT ANSWER: C

C - The work-energy theorem states that the work done is equal to the change of the kinetic energy of the particle,

$$W = \Delta K = K_f - K_i \qquad (1)$$

From the given data,

$$K_i = (1/2)mv^2 \qquad (2)$$

and

$$K_f = (1/2)m(2v)^2 = 2mv^2 \qquad (3)$$

Substituting (2) and (3) into (1), we find

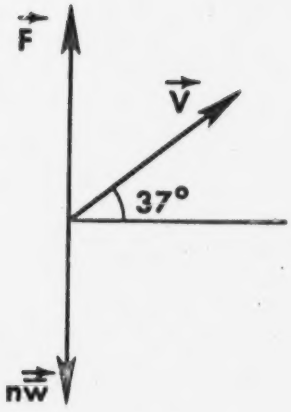
$$W = 2mv^2 - (1/2)mv^2 = (3/2)mv^2$$

[a] CORRECT ANSWER: A

A - The escalator must just overcome the gravitational force on the passengers (weight). If n is the number of passengers of average weight $\bar{w}$, the total force that must be provided by the escalator is $\vec{F} = -n\vec{\bar{w}}$. The power delivered by the escalator in moving the passengers with a velocity $\vec{v}$ is

$$P = \vec{F}\cdot\vec{v} = -n\vec{\bar{w}}\cdot\vec{v} = -n\bar{w}v\cos(90 + 37°)$$

$$= n\bar{w}v\sin(37°)$$



This power cannot be larger than

$$10 \text{ hp} \times 550 \frac{\text{ft-lb/sec}}{\text{hp}} = 5500 \text{ ft-lb/sec}$$

Therefore, for maximum n,

$$n\bar{w}v\sin(37°) = 5500$$

or

$$n = \frac{5500}{\bar{w}v\sin(37°)}$$

$$= 30.6$$

Hence, 30 people can ride the escalator. Notice that 31 people are too many.

TRUE OR FALSE? In the solution above, power consumed is independent of the angle of the escalator.

[b] CORRECT ANSWER: D

Circular motion provides an example of the dependence of the work done on the angle between the applied force and direction of motion. The centripetal force is mv^2/r, and the total distance traveled during one revolution is $s = 2\pi r = \pi d$. At any moment, however, the force is directed along the radius toward the center, while the direction of motion is along the tangent to the circle at the point at which the particle is. Thus, the angle between $\vec{F}$ and $d\vec{s}$ is 90°; and $\vec{F}\cdot d\vec{s} = Fds\cos(90°) = 0$. So the work done by the centripetal force is zero.

TRUE OR FALSE? In uniform circular motion, the work done by the centripetal force is an unpredictable function of displacement.

[a] CORRECT ANSWER: 10 m/sec

The kinetic energy K of the projectile is given by

$$K = \frac{1}{2} m(v_x^2 + v_y^2)$$

At the highest point in a trajectory, the vertical component of velocity vanishes,

$$K = \frac{1}{2} mv_x^2$$

the horizontal component of velocity remains unchanged throughout the projectile's flight because there is no horizontal force. The initial speed, v, and v_x are related by

$$v_x = v \cos\theta$$

so that

$$K = \frac{1}{2} m(v \cos\theta)^2$$

or

$$v = \frac{1}{\cos\theta}\sqrt{\frac{2K}{m}}$$

$$= 10 \text{ m/sec}$$

TRUE OR FALSE? Just before the projectile leaves the muzzle of the gun it has definite vertical and horizontal components of velocity.

[b] CORRECT ANSWER: 100 ft-lb

Since the displacement and time are given, and $v_o = 0$, the constant acceleration can be computed from

$$s = \frac{1}{2} at^2 \qquad \text{so} \qquad a = \frac{2s}{t^2}$$

The force causing this acceleration is

$$F = ma = \frac{2ms}{t^2}$$

Finally the work done by this force is given by

$$W = Fs = 2m \frac{s^2}{t^2} = 2 \times \frac{16}{32} \times \left(\frac{100}{10}\right)^2 = 100 \text{ ft-lb}$$

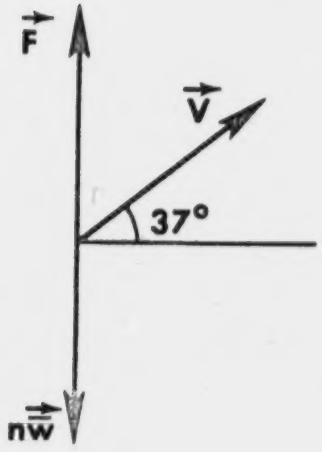
[a] This solution is identical to that of question 10, except for some numerical changes. The solution is reproduced here for your convenience.

CORRECT ANSWER: B

B - The escalator must just overcome the gravitational force on the passengers (weight). If n is the number of passengers of average weight $\bar{w}$, the total force that must be provided by the escalator is $\vec{F} = -n\vec{\bar{w}}$. The power delivered by the escalator in moving the passengers with a velocity $\vec{v}$ is

$$P = \vec{F}\cdot\vec{v} = -n\vec{\bar{w}}\cdot\vec{v} = -n\bar{w}v\cos(90 + 37°)$$

$$= n\bar{w}v\sin(37°)$$



This power cannot be larger than

$$5\text{ hp} \times 550\ \frac{\text{ft-lb/sec}}{\text{hp}} = 2250\text{ ft-lb/sec}$$

Therefore, for maximum n,

$$n\bar{w}v\sin(37°) = 2250$$

or

$$n = \frac{2250}{\bar{w}v\sin(37°)}$$

$$= 30.6$$

Hence, 30 people can ride the escalator. Notice that 31 people are too many.

TRUE OR FALSE? More people could have ridden the escalator if $\bar{w}$ had been smaller than 150 lb.

[b] CORRECT ANSWER: D

D - The work done on the book is given by $W = \vec{F}\cdot\vec{s} = F_s s$ where $\vec{F}$ is the applied force and $\vec{s}$ is the displacement. In this question $\vec{s} = \vec{y}$. Furthermore, in order to move the book vertically upward (without acceleration), a vertical force $\vec{F} = m\vec{g}$ is required to overcome the force of "gravity" (weight). Hence, $F_y = mg$ and

$$W = \vec{F}\cdot\vec{s} = F_y y = mgy$$

[a] CORRECT ANSWER: D

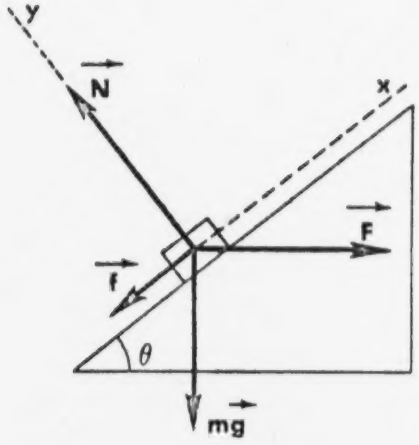


Figure 1

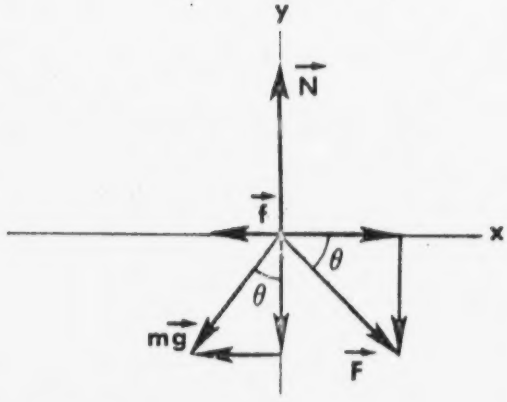


Figure 2

D - In Figure 1 we show all the forces acting on the block, along with the coordinate system chosen for this problem. The forces involved are:

i) the applied horizontal force $\vec{F}$,

ii) the block's weight $m\vec{g}$,

iii) the plane's reaction force $\vec{N}$, and

iv) the force of friction $\vec{f}$ of magnitude $f = \mu N$.

Since the displacement is along the positive x-axis, only the x-components of these forces will contribute to the work done on the block. Therefore, the normal reaction force $\vec{N}$ will not contribute directly to the work done, being perpendicular to $\vec{s}$. It does contribute indirectly, however, since the magnitude of the frictional force $\vec{f}$ is proportional to $\vec{N}$. The direction of $\vec{f}$ is, of course, always opposite to the motion.

The force $\vec{N}$ is

$$\vec{N} = -\Sigma \vec{F}_y$$

or written in scalar form,

$$N = -(-F \sin\theta - mg \cos\theta) = F \sin\theta + mg \cos\theta$$

next page

continued

So the magnitude of the force of friction becomes

$$f = \mu N = \mu(F \sin\theta + mg \cos\theta) \qquad (1)$$

The total work done on the block is (see Figure 2)

$$W = \vec{F}\cdot\vec{s} = s(\Sigma F_x) = s[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \qquad (2)$$

Using the work-energy theorem $\Delta K = W$, we find

$$\frac{1}{2} mv^2 = W$$

or

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2s}{m}\,[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)]} \qquad (3)$$

Substituting the given data in (3) we obtain

$$v = \sqrt{\frac{2 \times 10}{10}\,\{(120)(0.8) - (10)(9.8)(0.6) - (0.2)[(120)(0.6) + (10)(9.8)(0.8)]\}}$$

$$= \sqrt{2(96 - 58.8 - 30.08)} = \sqrt{14.24} = 3.76 \text{ m/sec}$$

TRUE OR FALSE? With the axes chosen as illustrated, the y-component of F does not directly contribute to the work done on the block.

[a] CORRECT ANSWER: 100 j

At its highest altitude, the vertical component of the projectile's velocity is momentarily zero. The horizontal component of the velocity is constant, since there is no horizontal force acting on the particle. Hence, the kinetic energy at the highest altitude is given by

$$K = \frac{1}{2} mv_x^2 = \frac{1}{2} m(v_o \cos\theta)^2 = \frac{1}{2} \times 2 \times (20 \times 0.5)^2 = 100 \text{ j}$$

TRUE OR FALSE? At the highest altitude, the projectile is accelerating uniformly in the horizontal direction.

[a] CORRECT ANSWER: C

C - The work done is found from

$$\int_{s_1}^{s_2} F_s ds \equiv \int_{x_1}^{x_2} F\, dx$$

with $F = -k(x - x_o)$. We have

$$W = -k\int_{x_1}^{x_2} (x - x_o)\, dx = -k\left(\frac{x^2}{2} - x_o x\right)\Bigg|_{x_1}^{x_2}$$

$$= -\frac{k}{2}\,(x_2^2 - 2x_o x_2 - x_1^2 + 2x_o x_1)$$

Substituting the values of $x_o = 0.15$ m, $x_1 = 0.10$ m, $x_2 = 0.25$ m, and $k = 2000$ nt/m above, we get

$$W = -1000(625 - 750 - 100 + 300) \times 10^{-4} = -7.5 \text{ j}$$

The negative sign means that work is done *on* the spring, not *by* the spring. We can see this qualitatively. The spring is originally compressed 5 cm from equilibrium. It does work in moving the mass from $x = 10$ cm to $x = x_o = 15$ cm. From $x = 15$ cm to $x = x_2 = 25$ cm, however, the spring must be stretched, so work must be done on it.

TRUE OR FALSE? Throughout the displacement from $x = 18$ cm to $x = 23$ cm, the nature of the work done is identified by a negative (-) sign.

[b] CORRECT ANSWER: 980 watts

The power delivered by a force can be written as

$$P = \vec{F}\cdot\vec{v}$$

In the present problem, the magnitudes of the applied force and the force of friction μN are equal, so $F = \mu N = \mu mg$. Furthermore, $\vec{F}$ and $\vec{v}$ have the same direction. Hence,

$$P = Fv = \mu mgv = (0.2)(100)(9.8)(5) = 980 \text{ watts}$$

[a] CORRECT ANSWER: 5 cm

We use the work-energy theorem to find the speed of the ball when it comes out of the water. Note that the force is in the direction of motion.

$$W = \Delta K$$

$$\bar{F}s = \frac{1}{2} mv^2 - \frac{1}{2} mv_o^2$$

where $\bar{F}$ is the magnitude of the average force and s is the distance traveled by the ball in water. Solving for the square of the final speed, we have

$$v^2 = \frac{mv_o^2 + 2\bar{F}s}{m}$$

The speed v is now the initial speed with which the ball leaves the surface of the water. The flight of the ball through the air is described by

$$v_f^2 = v^2 - 2gy$$

At the highest point, $v_f = 0$, so

$$y = \frac{v^2}{2g} = \frac{mv_o^2 + 2\bar{F}s}{2mg} = \frac{\bar{F}s}{mg} \quad \text{where } v_o \text{ has been set equal to zero}$$

$$= \frac{(490 \times 10^{-5})\ (10)}{(0.1)\ (9.8)} = 0.05 \text{ m} = 5 \text{ cm}$$

TRUE OR FALSE? In this solution, $v_o = 0$ because the initial velocity of the ball is zero when it is released from the bottom of the barrel.

[b] CORRECT ANSWER: 10 m/sec

The work W done by the block is equal to the change in the kinetic energy K of the block,

$$W = \Delta K$$

or

$$W = K_f - K_i$$

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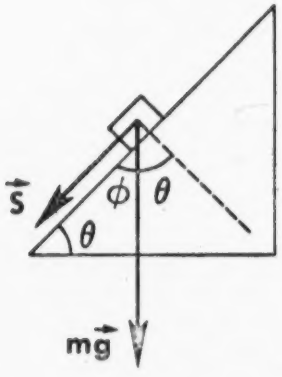


In this case, we have work done by the component of weight projected along the incline, mg sinθ

$$W = m\vec{g}\cdot\vec{s}$$

$$= mgs\cos\phi$$

$$= mgs\sin\theta$$

The work-energy theorem becomes

$$mgs\sin\theta = \frac{1}{2}m(v_f^2 - v_i^2)$$

or

$$v_f^2 = v_i^2 + 2gs\sin\theta$$

Using the given data v_i = 8 m/sec, s = 2.6 m and θ = 45°, we find v_f = 10 m/sec.

TRUE OR FALSE? All other things equal, at increasingly larger values of $\vec{s}$, the greater will be v_f.

[a] CORRECT ANSWER: C

Given that

$$F = ma = m(\alpha + \beta x^2)$$

the work done is

$$W = \int_{x_1}^{x_2}\vec{F}\cdot d\vec{s} = m\int_{x_1}^{x_2}(\alpha + \beta x^2)\,dx = m[\alpha x + \left(\frac{\beta}{3}\right)x^3]\Big|_{x_1}^{x_2}$$

$$= 2[5x + (15/3)\,x^3]\Big|_0^2 = 100\ \text{J}$$

TRUE OR FALSE? For the situation described, α and β are both constants.

[a] CORRECT ANSWER: B

B - The change in the block's kinetic energy is equal to the net work done on the block. The spring applies a force on the block up to the point the spring reaches its normal (equilibrium) length. This variable force is equal to

$$F = -k(x - x_o) \tag{1}$$

which is in the direction of the block's motion.

The work done by this force is

$$W_1 = \frac{1}{2} k(x - x_o)^2 \equiv \frac{1}{2} k s_1^2 \tag{2}$$

where s_1 is the change in the spring's length; namely,

$$x - x_o = s_1 = -20 \text{ cm} = -0.2 \text{ m}$$

the minus sign indicating a negative change of length, that is, compression. The work done by the force of friction is negative (work done against friction) and is equal to

$$W_2 = -\mu mg s_2, \text{ with } s_2 = \overline{AB} = 50 \text{ cm} = 0.5 \text{ m}. \tag{3}$$

Thus, at point B the block's kinetic energy is given by

$$\frac{1}{2} mv^2 = W_1 + W_2 = \frac{1}{2} k s_1^2 - \mu mg s_2 \tag{4}$$

Now, since to the right of B the surface is frictionless, the net force on the block there is zero, and the block will move with constant velocity. Therefore, the block's speed at point C is found from equation (4) and is

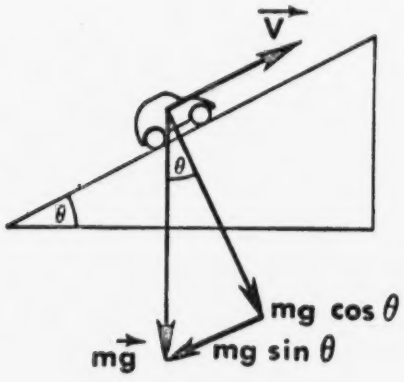
$$v = \sqrt{\frac{k}{m} s_1^2 - 2\mu g s_2} \tag{5}$$

Substituting the given data in (5) we find

$$v = \sqrt{\frac{573}{2} (0.2)^2 - 2(0.25)(9.8)(0.5)}$$

$$= \sqrt{11.45 - 2.45} = 3.00 \text{ m/sec}$$

[a] CORRECT ANSWER: 75,350. ft-lb/sec or 137 hp



The power can be computed from

$$P = \vec{F} \cdot \vec{v} \qquad (1)$$

The force $\vec{F}$ must be such as to overcome the weight of the car; i.e.,

$$\vec{F} = -m\vec{g} \qquad (2)$$

So,

$$P = -m\vec{g} \cdot \vec{v} = mgv \cos(90 + \theta)$$

$$= -mgv(-\sin\theta)$$

$$= mgv \sin\theta \qquad (3)$$

Equation (3) can also be derived by using the fact that mg sinθ is the component of the weight along the incline. The force provided by the car's engine must overcome this component. The component of the weight normal to the incline (mg cosθ) is counteracted by the road's "reaction" on the car.

The given speed is 60 mi/hr = 88 ft/sec. Also, in order to convert from ft-lb/sec to hp, we must divide (3) by 550 (1 hp = 550 ft-lb/sec). Thus,

$$P = \frac{(2500)(88)(0.342)}{550} = 137 \text{ hp}$$

[a] CORRECT ANSWER: 825 lb

The work-energy theorem can be used for solving this problem. From $W = \Delta K$ or $\overline{F}s = (1/2)mv^2$, we find

$$\overline{F} = \frac{mv^2}{2s} \qquad (1)$$

where $\overline{F}$ is the average force, s the distance over which $\overline{F}$ acts, and v is the initial velocity of the bullet. The quantities involved in equation (1) must be expressed in the appropriate units. We convert the bullet's weight into its mass

$$m = 1 \text{ oz} \times \frac{1}{16}\frac{\text{lb}}{\text{oz}} \times \frac{1}{32}\frac{\text{slug}}{\text{lb}} = \frac{1}{16 \times 32} \text{ slug}$$

The velocity v = 600 mi/hr = 880 ft/sec, and the penetration depth s = 11 inches = 11/12 ft. Thus,

$$\overline{F} = \frac{\frac{1}{16 \times 32} \times (880)^2}{2 \times 11/12} = \frac{(88)^2 \times 12 \times 10^2}{16 \times 32 \times 2 \times 11} = 825 \text{ lb}$$

UNITED STATES NAVAL ACADEMY

Self-Paced

Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Potential Energy and Conservative Forces

OBJECTIVE

To introduce the concept of potential energy and its place in conservative systems; to use these concepts in the solution of relevant problems.

Potential energy is the kind of energy that a body has by virtue of its position or configuration. When a body is raised to a higher level, it is able to do a certain amount of work when it falls back to its original height; this is potential energy of "position." Potential energy of configuration is illustrated by a stretched or compressed spring. Work must be done to change the configuration of the spring so that an equal amount of potential energy is stored in the spring under ideal conditions because it can do that amount of work in returning to its original length.

In either case, potential energy can be uniquely determined only by referring the change of position or the change of configuration to some particular reference which is arbitrarily assigned a value of zero. *Only a change of potential energy is significant.* Once, however, a position is determined to which a zero value of potential energy can be assigned, we may speak of the potential energy of a body. When a mass is lifted to a table top, the energy may be measured relative to the floor. If we had referred the energy to the cellar or attic, the potential energy at floor level, or at table level, would then have been different.

In its broadest sense, a force is conservative if the work it does in moving an object over any path back to its starting point is zero. A second important characteristic of the action of a conservative force is that the work it does in moving the object between two given points is independent of the path over which the object has been moved. Conservative forces are intimately related to potential and kinetic energy. When a conservative force does work on a body, this work is completely recoverable; indeed, this is the fundamental aspect of conservative systems.

In this section, you will be asked to

(a) recognize the differences between conservative and non-conservative forces;
(b) extend the work-energy theorem to situations involving conservative forces.

PROBLEMS

1. The work-energy theorem states that the work done by the resultant force on a particle is equal to the change in kinetic energy of the particle, $W = \Delta K$. *If* the resultant force is conservative, we also know that the total energy of the particle does not change, $\Delta K + \Delta U = 0$. In this case, which of the following statements is correct?

The work done by the resultant conservative force is equal to

A. the change in the potential energy of the particle, $W = \Delta U$

B. the change in the total energy of the particle, $W = \Delta E$

C. the negative of the change in the total energy of the particle, $W = -\Delta E$

D. the negative of the change in the potential energy of the particle, $W = -\Delta U$

2. Which of the following describes the action of a conservative force?

A. A block of wood slides down an inclined plane with uniform speed.

B. A cork is pushed under water and then released to bob up to the surface.

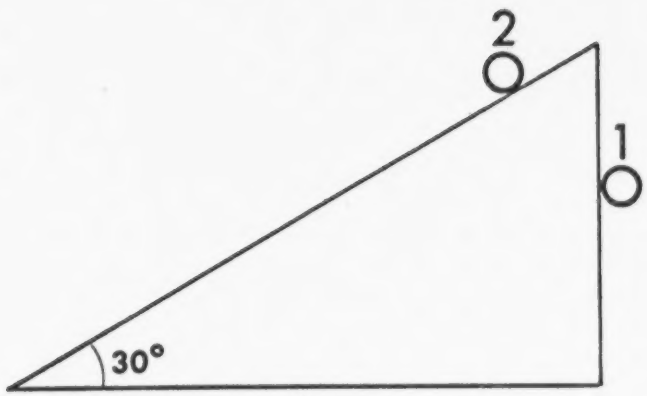
C. A meteor enters the atmosphere at high speed and succeeds in reaching the ground without burning up.

D. A rock is thrown vertically upward from the surface of the moon and allowed to fall back down.

3. Let the velocities of a particle at positions x_o and x be v_o and v, respectively. If the total energy E at x_o is $(1/2)mv_o^2 + U(x_o)$ and the particle is subjected to a conservative force, then for the position x, the expression $(1/2)mv^2 + U(x)$ is the equivalent of

A. $E + W$
B. $E + \Delta U$
C. $E + \Delta K$
D. E

4.



Two particles of equal mass are released from top of an incline making an angle of 30° with the horizontal. Particle one falls straight down and particle two slides down the incline, both reaching the same zero level. Neglecting friction, find the ratio of the work done by the gravitational force on particle one to the work done by the gravitational force on particle two.

A. 0.5
B. 2
C. 1
D. 0.866

INFORMATION PANEL Conservation of Energy

OBJECTIVE

To apply the principle of the conservation of energy to the solution of numerical problems in which it is directly involved.

The principle (or law) of conservation of energy states that in any isolated system, regardless of the changes that may occur within the system, the total energy of the system remains constant. Constancy implies that energy may be transferred from one part of the system to another, but that it cannot be created or destroyed. We know of no energy generators, only energy converters. The destruction of energy appears to be impossible. When energy seems to disappear, we always find that it has been merely transferred elsewhere and can always be accounted for.

next page

continued

In systems where *only conservative forces act*, the conservation principle may be written as:

$$K + U = \text{constant}$$

in which K = kinetic energy, U = potential energy, and the constant is called the total mechanical energy of the system. Any loss of kinetic energy that occurs results in a gain of an equal amount of potential energy, and vice versa so that

$$-\Delta U = \Delta K$$

$$\text{thus} \quad \Delta K + \Delta U = 0$$

When one or more nonconservative forces are present, the total mechanical energy E of the system is *not constant*, but changes by an amount equal to the work of the nonconservative force on the system.

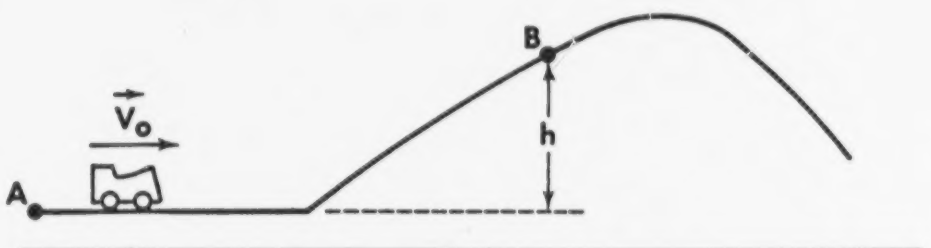
The problems associated with this section are all based on the conservation of mechanical energy, hence involve only conservative forces. You may assume for these that nonconservative forces are absent or that their effects may be ignored. You will be expected to be able to determine

(a) the maximum height of ascent of a vertically projected body with the help of the conservation principle;

(b) the height to which a roller coaster with a given initial speed will rise as it climbs an incline;

(c) determine the speed of a given pendulum bob as it passes through the lowest point of its swing.

5. A roller coaster moves at point A with speed v_o. At point B, the coaster moves with speed (1/2) v_o. Assuming no frictional losses, what is the height of point B above point A?



A. $3\ v_o^2/8g$

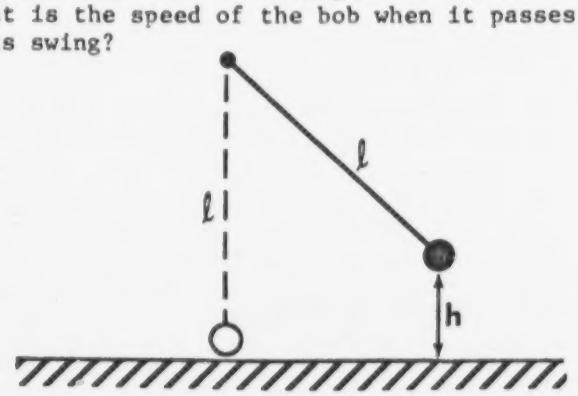
B. $7\ v_o^2/8g$

C. $v_o^2/4g$

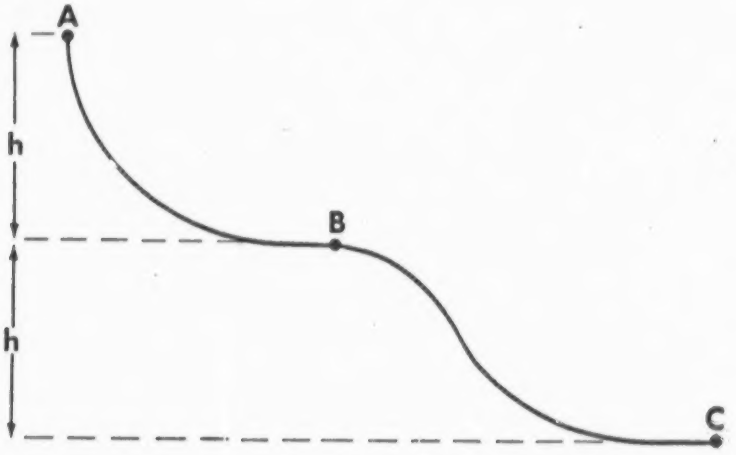
D. $5\ v_o^2/8g$

6. A ball of mass 0.5 kg is thrown from ground level vertically upward with a speed of 20 m/sec. Use conservation of energy to find the maximum height, h, attained by the ball.

7. A pendulum bob is released from a height h = 30 cm with a speed v_o = 2 m/sec. What is the speed of the bob when it passes through the lowest point of its swing?



8. A particle of mass m starts from rest at point A and slides down to point C without leaving the track. Neglecting friction, find the ratio of the speed of the particle at point B to its speed at point C, if the height of B above C is h and the height of A above C is 2h.



A. $\sqrt{2}$
B. 2
C. $\sqrt{1/2}$
D. 1/2

INFORMATION PANEL Potential Energy When the Resultant Force Varies

---

OBJECTIVE

To solve potential energy problems in which the force that produces the change in potential energy varies with the configuration of the system.

In one dimension, say, along the x-axis, the change in potential energy ΔU is related to the component of force along this axis $F(x)$ by the equation:

$$\Delta U = -\int_{x_0}^{x} F(x)\, dx$$

This relationship may also be written in the form:

$$F(x) = -\frac{d}{dx} U(x)$$

next page

continued

The second form is easily verified by substituting it in the first equation; an identity is obtained. Clearly, we can also write this last expression as:

$$dU(x) = -F(x)\,dx$$

In many situations, the force F varies with its position along the chosen axis, that is, F is a function of x. When you encounter a problem of this type, all that is required is that you set up the general equation you need and substitute the given identity for F(x). For example, suppose that you are told that the force is related to its position along the x-axis by:

$$F = kx^3$$

in which k is a constant. To find the change in the potential energy of a particle to which this force is applied, from some reference position x_o to a new position x, you would write:

$$dU = -(kx^3)\,dx$$

and then integrate between the positions x and x_o.

Generalized expressions for forces that vary in two and three dimensions are also derived in your text. In this section of your work, however, you will be dealing with one-dimensional problems only. You are expected to be able to solve problems in which you are to find

(a) the potential energy of a particle located at some arbitrary point on the axis being considered, given the way in which the force varies with position on this axis;

(b) the x-component of a force that varies in two dimensions, given the relationship between this force and the consequent potential energy it produces.

9. For a force

$$F = -ky$$

where k is a constant, and for the choice $U = 0$ at $y = y_o$, what is the potential energy U(y) of a particle located at an arbitrary point y?

10. The potential energy corresponding to a certain two-dimensional force is

$$U(x,y) = \frac{1}{3} k(x^3 + y^3)$$

where k is a constant. The x-component of the corresponding force is

A. $-k(x^2 + y^2)$
B. kx^2
C. $-kx^2$
D. $k(x^2 + y^2)$

11. A particle is subject to a force $F(y) = -mg$. If the potential energy of the particle is zero at the origin, $U(0) = 0$, what is the potential energy $U(y)$ of the particle as a function of y?

12. For a force

$$F(x) = -\frac{k}{x^2}$$

where k is a constant and for the choice of $U = 0$ at $x = \infty$, what is the potential energy $U(x)$ of a particle located at an arbitrary point x?

A. $-\frac{k}{x}$ B. $\frac{k}{x^2}$ C. $\frac{k}{x}$ D. kx^2

INFORMATION PANEL Energy in Springs

OBJECTIVE

To apply conservation principles to the solution of spring problems in which kinetic energy and potential energy are involved in ideal (frictionless) systems.

The problems found in this section require that you recall and apply the following fundamental relationships:

next page

continued

1. In an oscillating spring system isolated from external forces, the total energy remains constant and is always instantaneously equal to the sum of the kinetic energy and potential energy of the system at that instant, or:

$$E = K + U$$

2. The force required to compress or stretch a spring is proportional to the compression or extension, i.e.,

$$F_{app} = kx,$$

where k is defined as being the spring constant. The potential energy of the spring is therefore

$$U = \frac{1}{2} kx^2$$

in which k is the spring constant and x is the displacement of the end of the spring from its zero reference position.

3. When a spring is compressed or stretched as a result of the transfer of energy to it from a moving mass m having a velocity v, the conservation principle may be written:

$$E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

where E is the constant, total energy of the mass-spring system.

In this section, you will be asked to solve problems in which you must determine:

(a) the height from which a mass must be dropped onto a spring in order to produce a given compression;

(b) the kinetic energy of a mass on a vibrating spring when the total energy and potential energy at a given instant are known;

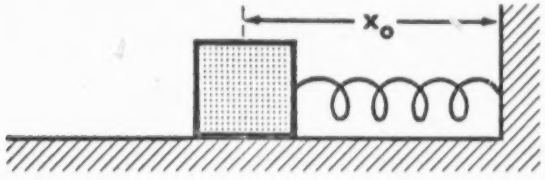
(c) the maximum displacement of a mass on a vibrating spring given the spring constant and the total energy of the system;

(d) the maximum compression of a given spring after being struck by a given mass moving at a given speed.

13. A ball of mass m is dropped from rest onto a spring with spring constant k. The maximum compression of the spring is x. Find the height above the (uncompressed) spring from which the ball was dropped, assuming no friction at the time of impact.

A. $(kx^2/2mg) - x$
B. $(kx^2/2mg) + x$
C. $(kx^2/mg) - x$
D. kx^2/mg

14. A mass is attached to a spring as shown in the diagram. The total energy of the mass-spring system is 1.28 j. Assume that the system is frictionless and find the kinetic energy for the instant at which the potential energy is 0.5 j.



15. For the mass-spring system of the foregoing problem, what is the maximum displacement of the mass from its equilibrium position x_o if the spring constant is k = 4 nt/m and the total energy E = 1.28 j?

16. A block of mass m slides along a frictionless horizontal surface with speed v. The block is stopped by compressing a massless spring with spring constant k. The maximum compression is

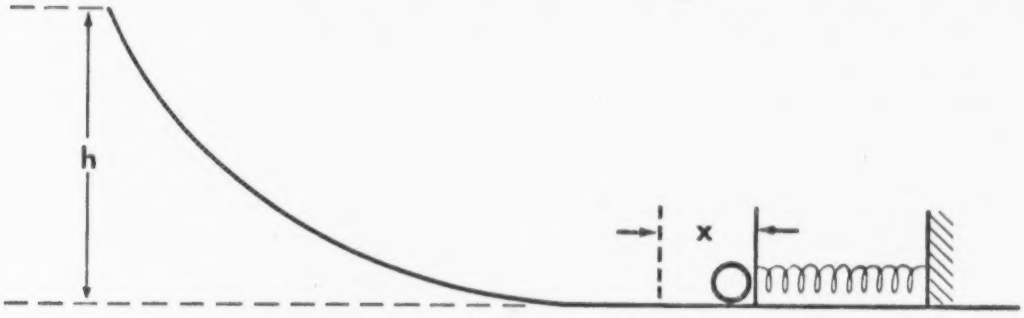
A. $\sqrt{m/2k}\ v$

B. $\sqrt{m/k}\ v$

C. $\sqrt{2m/k}\ v$

D. $\sqrt{m/k}\ v$

17. A particle of mass m is released from the top of an incline as shown in the figure below. At the bottom, it compresses a spring by an amount indicated as x. The spring constant is k. Find the height h from which the particle is released. Neglect function.



A. $\frac{2mg}{kx^2}$ B. $\frac{kx^2}{2mg}$ C. $\frac{kx^2}{mg}$ D. $\frac{mg}{kx^2}$

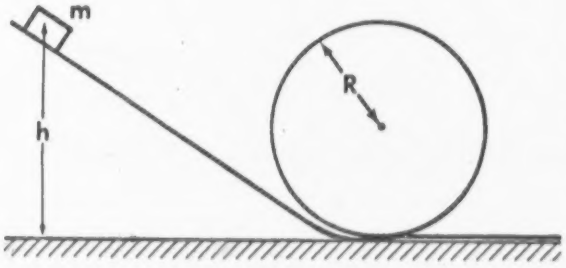
INFORMATION PANEL A Composite Problem Using Conservation of Energy

---

OBJECTIVE

To solve a problem in which the principle of conservation of energy is combined with centripetal force.

In the system illustrated in the accompanying diagram, a block is placed on a frictionless inclined track and released so that it slides down the track into an inside loop. If it is not placed high enough above the reference surface, it will start the loop but fall off before it reaches the top. If it is to successfully negotiate the loop and continue on its way, there is a definite minimum height at which it must be placed before it is released. Let's talk about this problem with a view to helping you get started on it.



You are given the mass or weight of the block and the radius of the loop. When the block passes the bottom of the loop just as it comes off the incline, it is moving at some specific speed, but the speed must decrease as it begins to climb the far end of the loop against gravity. The loop

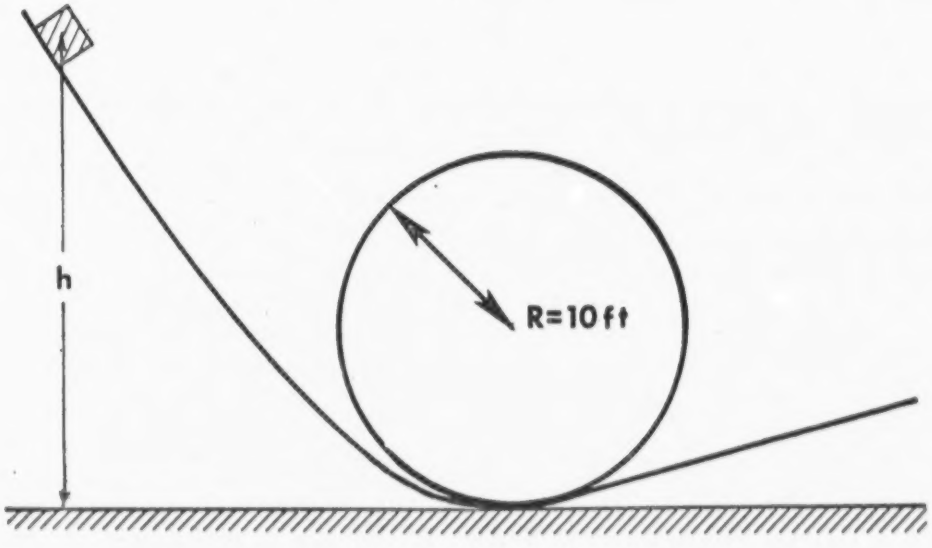
next page

continued

is rigid and presses inward on the block, providing the centripetal force needed to force it into circular motion. The lowest velocity to which the block will be reduced is the velocity at the very top of the loop. What physical situation must obtain at this point if the block is not to leave the track, falling downward? Think about this before continuing.

If the velocity of the block at the top of the loop is less than a certain critical value, its weight will be greater than the centripetal force required to keep it moving in a loop of the given radius. In this case, it will simply leave the track and fall back to the surface. Thus, if it is not to lose contact with the track, its velocity must be greater than this critical value. Stated otherwise, *its velocity must be such that the centripetal force needed to make it move in a circle of that particular radius is equal to or greater than the weight of the block.* This is all the clue you should need to get started. Set up the expression for centripetal force in terms of mass and velocity, then equate this with the weight expressed in terms of mass and gravitational acceleration. Look for a way to get the kinetic energy of the block into the picture after you have done this.

18.



Compute the minimum height h from which a 10-lb block can be released, in order that it will go around the loop without losing contact with the track. Assume a frictionless track.

[a] CORRECT ANSWER: 0.78 j

The system is conservative and the principle of conservation of energy holds:

$$E = K + U$$

where total energy E is a constant. Substituting the data 1.28 j = K + 0.50 j, we obtain K = 0.78 j.

[b] CORRECT ANSWER: 3.14 m/sec

If we take the lowest point as the zero-potential energy point, the bob's total energy at release is $E_i = (1/2)mv_o^2 + mgh$. This must be equal to its total energy when it passes through the lowest point of its swing, $E_f = (1/2)mv^2$. Equating E_i and E_f and solving for v, we obtain

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 + mgh$$

or

$$v = \sqrt{v_o^2 + 2gh}$$

$$= \sqrt{(2)^2 + 2(9.8)(0.3)}$$

$$= 3.14 \text{ m/sec}$$

Notice that this is the result when the pendulum bob is initially moving either clockwise or counterclockwise.

[c] CORRECT ANSWER: C

The gravitational force is conservative, therefore, the work done is equal to negative of the change in potential energy. In other words,

$$\Delta W = -\Delta U$$

In this problem the ΔU is the same for both the particles, therefore, the ratio of the work done is *one*.

TRUE OR FALSE? To arrive at the statement $W = -\Delta U$, one must apply *both* the principle of conservation of energy and the work-energy theorem.

[a] CORRECT ANSWER: A

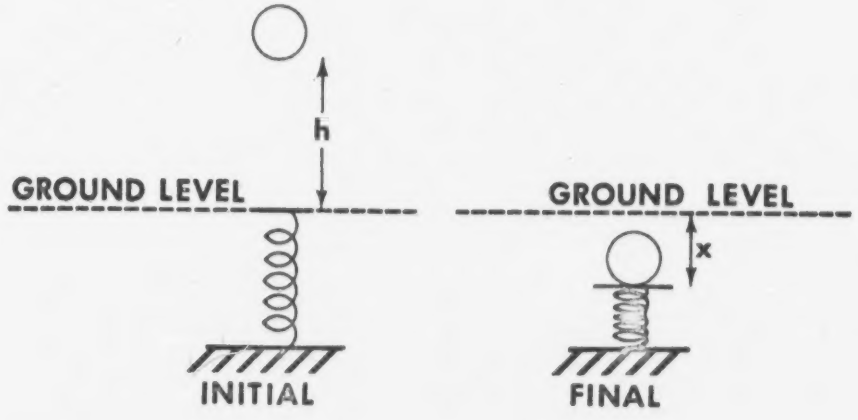
Initially, the kinetic energy of the ball is zero. If we define our zero of gravitational potential energy to be the "ground" level in the accompanying diagram, the potential energy of the ball is mgh, where h is the unknown height above the spring. The uncompressed spring has zero potential energy. Therefore, the total initial energy is

$$E_i = U_i + K_i = mgh + 0 = mgh \qquad (1)$$

In the final configuration, the ball is momentarily at rest (zero kinetic energy). The potential energy due to gravity is mg(-x) because the ball is below the "ground" level. The spring now contributes a potential energy of $(1/2)kx^2$ due to its compression. We have

$$E_f = U_f + K_f = [mg(-x) + (1/2)kx^2] + 0$$

$$= -mgx + (1/2)kx^2 \qquad (2)$$



Conservation of energy, $E_i = E_f$, gives

$$mgh = -mgx + (1/2)kx^2$$

or

$$h = \frac{kx^2}{2mg} - x$$

TRUE OR FALSE? As it turns out, the height from which the ball is dropped is directly proportional to the compression of the spring.

[a] CORRECT ANSWER: B

Applying the law of conservation of energy:

$$\frac{1}{2}mv^2 = mgh \qquad (1)$$

where v is the speed of the particle on the flat part of the track. Similarly, the energy relationship for the particle and spring system is:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \qquad (2)$$

Combining (1) and (2) and solving for h yields

$$h = \frac{kx^2}{2mg}$$

TRUE OR FALSE? After the ball passes the bottom of the incline and while it moves along the flat part of the track, its acceleration is zero.

[b] CORRECT ANSWER: D

The force of gravity is a conservative force. Since the moon has no atmosphere, friction is not operative during the rock's flight. When the rock returns to the height from which it was thrown, it will have its initial speed and the kinetic energy will also have its initial value.

In general we look for dissipative forces (usually frictional forces), which will convert some, or all, of the mechanical energy into thermal energy (heat), light, or sound which, for practical purposes, is not recoverable. Such forces are non-conservative.

When a rock is thrown near the surface of the Earth, the process is not conservative unless air resistance is neglected. Although the gravitational force is conservative, the force due to friction is dissipative and the resultant force is therefore dissipative. Answers A, B, and C involve frictional forces and are therefore not descriptive of conservative forces.

[a] CORRECT ANSWER: C

Using the principle of conservation of mechanical energy, the speed at points B (v_B) and the speed at point C (v_C) may be computed from

$$\frac{1}{2} mv_B^2 = mgh \qquad (1)$$

and

$$\frac{1}{2} mv_C^2 = mg(2h) \qquad (2)$$

Therefore the ratio

$$\frac{v_B}{v_C} = \sqrt{\frac{1}{2}}$$

TRUE OR FALSE? At point B, the kinetic energy is smaller than the total energy of the particle at that instant.

[b] CORRECT ANSWER: D

The forces are conservative, so the total energy at x is equal to the total energy at x_o. Hence,

$$\text{Total energy at } x_o = \frac{1}{2} mv_o^2 + U(x_o) = E$$

so that

$$\text{total energy at } x = \frac{1}{2} mv^2 + U(x) = E$$

Hence, the total energy at x = E. Clearly, answers A, B, C all have an incorrect additive on the right side of the equation.

[c] CORRECT ANSWER: D

When conservative forces are involved, the total energy is conserved. Thus, $\Delta E = \Delta K + \Delta U = 0$, and $\Delta K = -\Delta U$. Then, using the work-energy theorem, we get,

$$W = \Delta K = -\Delta U$$

TRUE OR FALSE? The statement $\Delta K + \Delta U = 0$ is the equivalent of the statement $K + U$ = constant for a conservative system.

[a] CORRECT ANSWER: 20.4 m

We choose our zero level of potential energy at the position from which the ball was thrown. Its total energy at the instant it leaves the ground consists entirely of its initial kinetic energy, $(1/2)mv_o^2$. Similarly, at the instant the ball attains its maximum height its total energy is potential, mgh. Using conservation of energy we obtain

$$mgh = \frac{1}{2} mv_o^2$$

which gives

$$h = \frac{v_o^2}{2g} = \frac{(20)^2}{2 \times 9.8} = \frac{400}{19.6} = 20.4 \text{ m}$$

In general, for a single particle in a gravitational field, equating initial energy to final energy yields

$$mgy_o + \frac{1}{2} mv_o^2 = mgy + \frac{1}{2} mv^2$$

from which

$$v^2 = v_o^2 - 2g(y - y_o)$$

This equation should be familiar to you from kinematics.

[b] CORRECT ANSWER: 0.8 m

At either end of the swing ($x = x_{max}$) the speed of the mass is momentarily zero, so the total energy of the mass-spring system (1.28 j) is potential. Thus,

$$U = \frac{1}{2} k(x_{max} - x_o)^2 = E = 1.28 \text{ j}$$

and the distance from the equilibrium position

$$s = x_{max} - x_o = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(1.28)}{4}} = \sqrt{0.64} = 0.8 \text{ m}$$

[a] CORRECT ANSWER: 25 ft

At the top of the loop the block must have a velocity such that the centripetal force required to make it move in a circle is at least as great as the weight of the block. It can, of course, be greater, since the track can provide all the additional force required. We must, then, have

$$\frac{mv^2}{R} \geq mg$$

or

$$\frac{mv^2}{2} \geq \frac{mgR}{2} \qquad (1)$$

But from conservation of energy (no friction)

$$\frac{mv^2}{2} = mg(h - 2R) \qquad (2)$$

Combining (1) and (2) we obtain

$$mg(h - 2R) \geq \frac{mgR}{2}$$

and

$$h \geq 2R + \frac{R}{2} = \frac{5R}{2}$$

or

$$h \geq 25 \text{ ft}$$

TRUE OR FALSE? When the block is at the highest point of the loop, the centripetal force must be greater or equal to the weight of the block.

[a] CORRECT ANSWER: B

The initial state consists of a moving block with kinetic energy $(1/2)mv^2$, and an uncompressed spring with zero potential energy.

The total energy initially is

$$E_i = \frac{1}{2} mv^2$$

The final configuration consists of the block momentarily at rest, and the spring compressed an unknown distance x. We have

$$E_f = \frac{1}{2} kx^2$$

Now since energy is conserved, $E_i = E_f$, which gives

$$x^2 = \frac{m}{k} v^2 \quad \text{or} \quad x = \sqrt{\frac{m}{k}}\, v$$

[b] CORRECT ANSWER: A

Mechanical energy is conserved. Taking the initial potential energy to be zero, we have $E = (1/2)mv_o^2$. Equating this to the total final energy, kinetic plus potential, we obtain

$$\frac{1}{2} mv_o^2 = \frac{1}{2} m \left(\frac{v_o}{2}\right)^2 + mgh$$

The resulting expression for h is

$$h = \frac{(1/2)v_o^2 - (1/8)v_o^2}{g} = \frac{3v_o^2}{8g}$$

TRUE OR FALSE? The initial total energy of the roller coaster is partly kinetic and partly potential.

[a] CORRECT ANSWER: $U(y) = mgy$

In one dimension we have

$$F(y) = - \frac{dU(y)}{dy}$$

or

$$dU = - F(y)\, dy \qquad (1)$$

When integrating equation (1) remember that

$$\int_{U(0)}^{U(y)} dU = - \int_0^y F(y)\, dy = \int_0^y mg\, dy$$

or

$$U(y) - U(0) = - mgy \Big|_0^y = mgy$$

Since $U(0) = 0$,

$$U(y) = mgy$$

[b] CORRECT ANSWER: C

The force in two dimensions is given by

$$\vec{F} = - \frac{\partial U}{\partial x} \vec{i} - \frac{\partial U}{\partial y} \vec{j} \qquad (1)$$

In the computation of $\partial U/\partial x$, y is treated as though it were a constant. We have

$$F_x = - \frac{\partial}{\partial x} \left[\frac{1}{3} k(x^3 + y^3) \right]$$

but since $\partial x^3/\partial x = d(x^3)/dx = 3x^2$

and $\partial(y^3)/\partial x = 0$

then $F_x = - \frac{1}{3} k(3x^2) = -kx^2$, the answer to the problem.

[a] CORRECT ANSWER: $\frac{1}{2}\, k(y^2 - y_o^2)$

From $$F_y = - \frac{\partial U}{\partial y} = - \frac{dU(y)}{dy}$$

we find $$dU = - F\, dy$$

Integrating between the points y_o and y,

$$\int_{U(y_o)}^{U(y)} dU = - \int_{y_o}^{y} F\, dy$$

Using the given data $F = - ky$ and $U(y_o) = 0$, we find

$$U(y) = \frac{ky^2}{2} \Bigg|_{y_o}^{y} = \frac{1}{2}\, k(y^2 - y_o^2)$$

TRUE OR FALSE? According to our established symbolism, the force F in this problem is confined to one dimension.

[b] CORRECT ANSWER: C

From $$F(x) = - \frac{\partial U}{\partial x} = - \frac{dU}{dx}$$

we find $$dU = - F(x)\, dx = \frac{k}{x^2}\, dx.$$

Integrating from the points $x = \infty$ to $x = x$

$$\int_{U(\infty)}^{U(x)} dU = \int_{\infty}^{x} \frac{k}{x^2}\, dx$$

$$U(x) = \frac{k}{x}$$

Note that $U(\infty) = 0$

TRUE OR FALSE? The potential energy of the particle as x approaches ∞ is less than k but greater than zero.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

The Coordinates of the Center of Mass of a System of Particles

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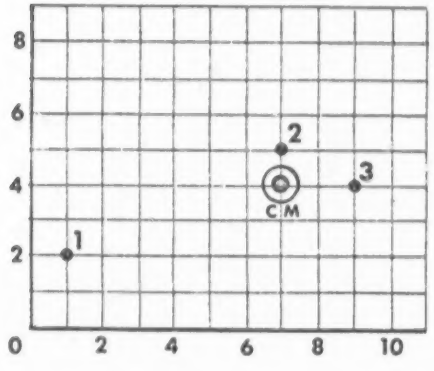
OBJECTIVE

To recognize and apply the fact that the position of the center of mass of a system of particles is independent of the coordinates used to describe its location.

Your assigned reading will ultimately carry you to the statement:

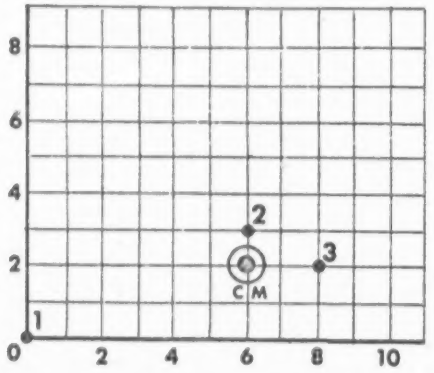
> *The center of mass of a system of particles depends only on the masses of the particles and the position of the particles relative to one another.*

One important implication of this statement is that the orientation or position of the axes does not affect the position of the center of mass relative to the particles. Consider the three particles in the upper diagram at the left. These particles have relative masses of 1, 2, and 3 respectively as indicated. Using the accepted method for determining the coordinates of the center of mass, we can write:



$$\bar{x}_{cm} = \frac{(1)(1) + (2)(7) + (3)(9)}{1 + 2 + 3} = 7$$

$$\bar{y}_{cm} = \frac{(1)(2) + (2)(5) + (3)(4)}{1 + 2 + 3} = 4$$



In the lower diagram, we have redrawn the system with a new set of axes so that the particle of relative mass 1 is now at the origin. Once again writing the coordinates of the center of mass, we have

$$\bar{x}'_{cm} = \frac{(1)(0) + (2)(6) + (3)(8)}{1 + 2 + 3} = 6$$

$$\bar{y}'_{cm} = \frac{(1)(0) + (2)(3) + (3)(2)}{1 + 2 + 3} = 2$$

next page

continued

Compare the two diagrams and observe that the center of mass in each one is identically located *relative to the system of particles*. Despite the fact that the particle of mass 1 is squarely on the origin and so has coordinates of (0,0), it still enters into the calculation of the center of mass. Note that it *does* contribute to the denominator of the fraction although it drops out of the numerator in both the evaluations.

To handle the problems in this group, it will be necessary for you to

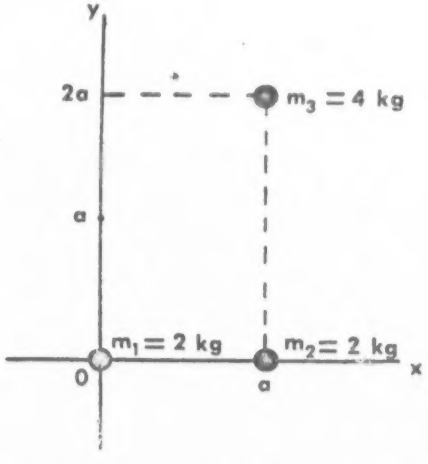
(a) locate the center of mass of three particles, two of which lie on the x-axis;

(b) write an expression for the x-coordinate of the center of mass of two particles in general form;

(c) determine the coordinates of the center of mass of an asymmetrical body in two dimensions.

PROBLEMS

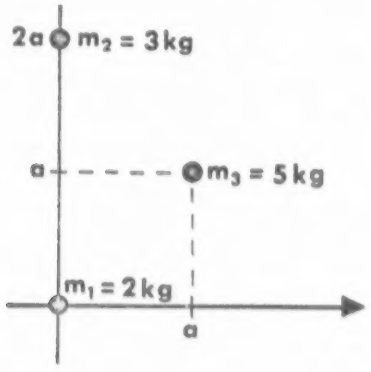
1. The coordinates of the center of mass of the system shown in the figure are



A. x = a; y = 1.33 a
B. x = 0.25 a; y = a
C. x = a; y = 0.75 a
D. x = 0.75 a; y = a

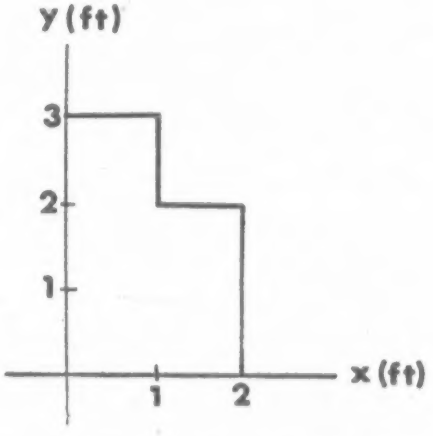
2. Consider the center of mass of a system of two particles m_1 and m_2 lying along the x-axis at x_1 and x_2, respectively. Write an expression for x_{cm}, the x-coordinate of the center of mass of the two particles.

3. What are the coordinates of the center of mass of the system shown in the figure?



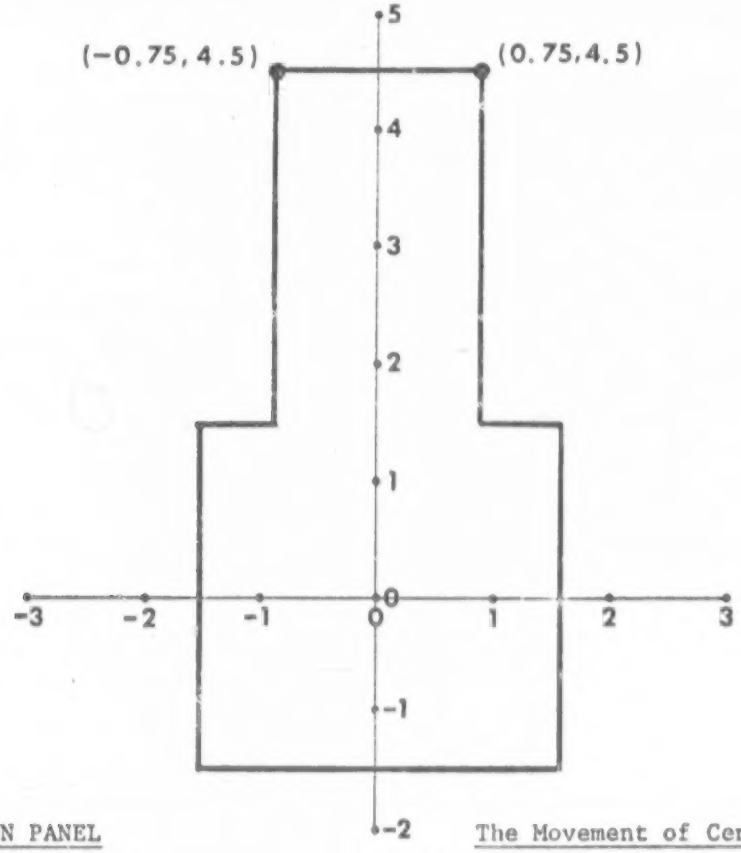
A. x = 2a; y = a
B. x = 0.5 a; y = 1.1 a
C. x = a; y = a
D. x = 0.33 a; y = a

4. A piece of 3/4 inch plywood has been cut into the shape shown. If uniform mass density and thickness are assumed for this piece of wood, then the center of mass is located at the point



A. (0.9, 1.0)
B. (1.3, 1.3)
C. (0.9, 1.3)
D. (1.0, 1.3)

5. A piece of 1/2 inch plywood has been cut into the shape shown. If uniform mass density and thickness are assumed for this piece of wood, what are the coordinates of its center of mass?



INFORMATION PANEL — The Movement of Center of Mass

OBJECTIVE

To correctly analyze problems involving the movement of the center of mass of a system of particles subjected to internal and/or external forces.

Since the basic definition of center of mass commits us to thinking of it as a point where all the mass of a body may be considered to be concentrated, it follows from Newton's second law that

> *The center of mass of a system of particles is a point which moves as though the total mass of the system is concentrated at that point and is subject to a force equal to the resultant of all* external *forces on the system.*

next page

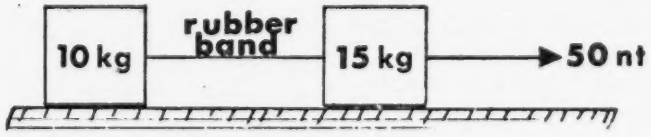
continued

Suppose a perfectly symmetrical hollow ball containing a compressed spring is made up of two segments fitted together. Suppose further that the ball is dropped vertically downward from the top of a bridge and that the spring is timed to decompress after the ball has fallen part of the way down, blowing the segments apart. The force exerted by the spring on the segments is an *internal* one; it must be accompanied by a reaction force which makes the net force on the segment system zero. All internal forces have this characteristic, hence the emphasis on *external* forces in the italicized statement above. Thus, internal forces cannot affect the motion of the center of mass of any system of moving particles. In this example, the two fragments of the ball would follow trajectories such that the center of mass of the two-fragment system would continue to *fall straight down as though the ball had not blown apart at all*. Note that no mention was made in the description about the relative masses of the two fragments. The apparent disregard of the center of mass for internal forces applies equally well to a pair of fragments having a mass ratio of, say, 10 to 1 as it does to a pair of fragments of equal mass. The only external force acting on the system--whether intact or segmented--is the downward force of gravitation so that the center of mass will have an acceleration equal to g just as the whole ball would have had if it had not come apart.

It is left as a thought exercise for you to visualize the difference in flight pattern of two equal-mass fragments as compared with two fragments of unequal mass.

All of the problems in this set require that you be able to recognize the differences in effect of internal and external forces acting on a system of particles.

6. Two masses on a table are connected by a rubber band. A constant force of 50 nt is applied to the right mass as shown. The coefficient of kinetic friction between each mass and the table is $\mu = 0.2$. The left mass is 10 kg and the right mass is 15 kg. What is the acceleration of the center of mass when both masses are moving to the right?



7. A weightlifter's barbell is accidentally released from the cargo hatch of a moving airplane. One of the weights separates from the bar in midair. Which statement best describes the resulting motion?

A. The center of mass of the whole barbell system will follow the same trajectory as the one it would follow if the weights did not separate.

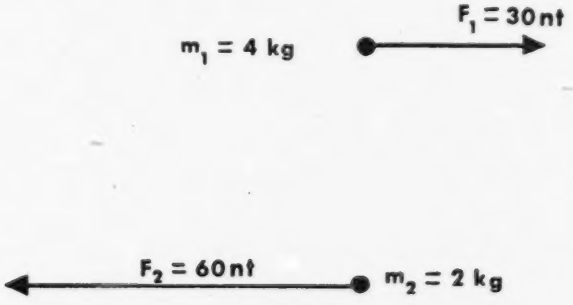
B. The motion of the pieces is completely random.

C. The pieces follow a path such that their center of mass falls to the ground along a straight vertical line.

D. The motion cannot be described because there is insufficient data.

8. A shell explodes in mid-trajectory near the surface of the Earth. Neglecting friction, name the geometric curve which describes the trajectory that the center of mass of the exploded shell will follow while both fragments are in flight.

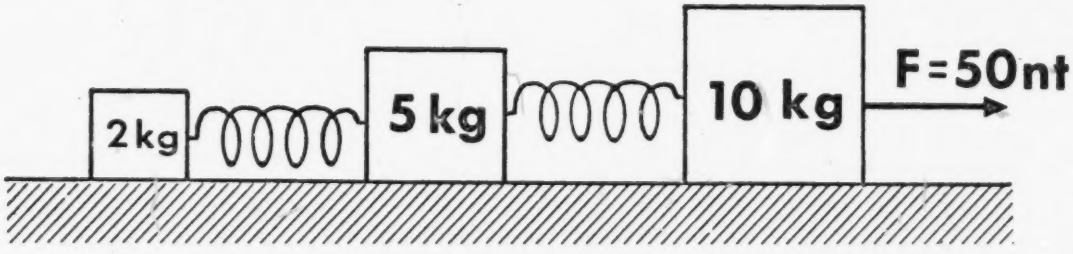
9.



For the system of masses and forces shown above, the acceleration of the center of mass is

A. 22.5 m/sec^2 towards left
B. 37.5 m/sec^2 towards right
C. 5 m/sec^2 towards left
D. 15 m/sec^2 towards right

10. Three masses on a table are connected by springs as shown in the figure below. A constant force of 50 nt is applied to the extreme right mass as shown. The coefficient of kinetic friction between each mass and the table is m = 0.2. The masses of the blocks are 2 kg, 5 kg and 10 kg as shown in the diagram. What is the acceleration of center of mass when all the masses are moving to the right?



[a] CORRECT ANSWER: Parabola

In this situation, the net external force on all the particles is equal to the weight of the whole system. Therefore, the center of mass of the fragmented shell moves as though it were the center of mass of the *intact* shell. Its trajectory will be parabolic because the internal explosive force cannot affect the motion of the center of mass.

[b] CORRECT ANSWER: 4 cm/sec^2

This problem looks much more difficult than it is. The acceleration of the center of mass is just the net external force divided by the total mass. The total mass is 25 kg. The net external force is the applied external force of 50 nt minus the frictional force exerted by the table. The total frictional force is

$$f = \mu N = \mu mg = \mu mg(m_1 + m_2) = 0.2 \times 9.8 \times 25 = 49 \text{ nt}$$

Hence, the net external force is 50 - 49 = 1 nt, from which we obtain an acceleration

$$a = \frac{1}{25}\frac{nt}{kg} = 0.04 \text{ m/sec}^2 = 4 \text{ cm/sec}^2$$

The answer is independent of the speed of either mass (both moving to the right). If either mass were stationary or moving in the opposite direction, the frictional forces would be different from those calculated above. For moving blocks, we have to take into account that the frictional force is always directed opposite to the velocity. If a block is stationary, we must use μ_s instead of μ_k in our calculations.

Note that the forces exerted by the rubber band on the masses are internal forces. They must be taken into account when free-body diagrams are drawn for the two masses, but they do not affect the motion of the center of mass.

TRUE OR FALSE? The rubber band may be replaced by a massless string without changing the acceleration of the center of mass.

[a] CORRECT ANSWER: B

By definition

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} \qquad \text{and} \qquad y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

Therefore

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (3)(0) + (5)(a)}{2 + 3 + 5} = 0.5a$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (3)(2a) + (5)(a)}{2 + 3 + 5} = 1\ 1a$$

Notice that although m_1 does not contribute to $\Sigma\ m_i \vec{r}_i$ (since $\vec{r}_1 = 0$), it must be included in $\Sigma\ m_i$.

TRUE OR FALSE? If m_1 is made 1 kg instead of 2 kg, it may then be omitted from the summations.

[b] CORRECT ANSWER: A

Before the pieces separated there were internal forces exerted by the pieces on each other. After the separation there is no interaction between the separate pieces. However, as far as Newton's second law is concerned, the situation has not changed. The forces before separation were internal, action-reaction forces and by Newton's third law their resultant was zero. Therefore, the net external force on the whole system has not changed with the separation.

[a] CORRECT ANSWER: (0.0, 1.0)

Divide the given piece of wood into two pieces: one square of area 9 ft^2 and another rectangle of area 4.5 ft^2. The centers of mass of these pieces are located at the points $c_1 = (0,0)$ for the square and $c_2 = (0,3)$ for the rectangle. Since the plywood is of uniform thickness, the masses of these two pieces are $m_1 = 9\rho$ and $m_2 = 4.5\rho$ where ρ is the assumed mass per ft^2 and m_1 and m_2 are the masses of the square and the rectangle, respectively. The total mass of the given piece is $M = 13.5\rho$. The problem now has been reduced to finding the center of mass of two particles of mass m_1 and m_2 located at c_1 and c_2 respectively.

$$x_{cm} = \frac{(9\rho \times 0.0 + 4.5\rho \times 0.0)}{13.5\rho} = 0.0 \text{ ft}$$

$$y_{cm} = \frac{(9\rho \times 0.0 + 4.5\rho \times 3)}{13.5\rho} = 1.0 \text{ ft}$$

TRUE OR FALSE? In a problem of this type, the position of the center of mass can be described only by giving at least three coordinates.

[b] CORRECT ANSWER: $(m_1x_1 + m_2x_2)/(m_1 + m_2)$

The mass-weighted mean of the positions of n particles is the center of mass of the system. The x-coordinate of the center of mass x_{cm} is given by

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Thus, for the two particles in this question

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

[a] CORRECT ANSWER: D

By definition,

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} \qquad \text{and} \qquad y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

So,

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (2)(a) + (4)(a)}{2 + 2 + 4} = 0.75\ a$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (2)(0) + (4)(2a)}{8} = a$$

Notice that, although m_1 does not contribute to $\Sigma\ m_i \vec{r}_i$ (since $\vec{r}_1 = 0$), it must be included in $\Sigma\ m_i$.

TRUE OR FALSE? One of the 2-kg masses has a y-coordinate equal to a.

[b] CORRECT ANSWER: 0.98 m/sec^2

The acceleration of the center of mass is the net external force divided by the total mass. The total mass is 17 kg. The net external force is the applied external force of 50 nt minus the frictional force exerted by the table. The total frictional force is

$$F = \mu N = \mu g(m_1 + m_2 + m_3)$$

$$= 0.2 \times 9.8\ \text{m/sec}^2 \times 17\ \text{kg}$$

$$= 33.32\ \text{nt}$$

Therefore, the acceleration is

$$a_{cm} = \frac{50\ \text{nt} - 33.32\ \text{nt}}{17\ \text{kg}} = 0.98\ \text{m/sec}^2$$

TRUE OR FALSE? In this problem, the friction due to air resistance has been ignored.

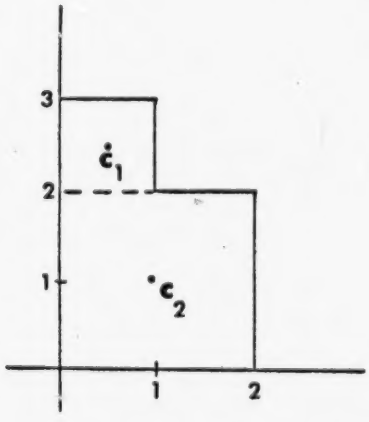
[a] CORRECT ANSWER: C

The center of mass of a system moves in the same way that a single particle of equal mass subject to the same external force would move. The resultant external force is F = 30 nt to the left and the total mass is $m_1 + m_2 = 6$ kg. Therefore, the acceleration of the center of mass is

$$\vec{a}_{cm} = \frac{\vec{F}}{m} = \frac{-30\ \hat{i}}{6}\ \frac{nt}{kg} = -5\ \hat{i}\ nt/kg = 5\ m/sec^2 \text{ toward left}$$

[b] CORRECT ANSWER: C

We shall present two ways of solving this problem. One way is to divide the given piece of wood into two squares of area 1 ft^2 and 4 ft^2, respectively. The centers of mass of these square pieces are located at the points $C_1 = (0.5, 2.5)$ and $C_2 = (1,1)$. Since the plywood is of uniform density and thickness, the masses of these two square pieces are respectively $m_1 = 1\rho$ and $m_2 = 4\rho$, where ρ is the assumed mass-per-unit-area. The total mass of the given piece is $M = 5\rho$. The problem now has been reduced to finding the center of mass of two particles of mass m_1 and m_2 located at C_1 and C_2, respectively.



$$x_{cm} = \frac{1}{5\rho}(\rho \times 0.5 + 4\rho \times 1) = \frac{0.5 + 4}{5} = 0.9 \text{ ft}$$

and

$$y_{cm} = \frac{1}{5\rho}(\rho \times 2.5 + 4\rho \times 1) = \frac{2.5 + 4}{5} = 1.3 \text{ ft}$$

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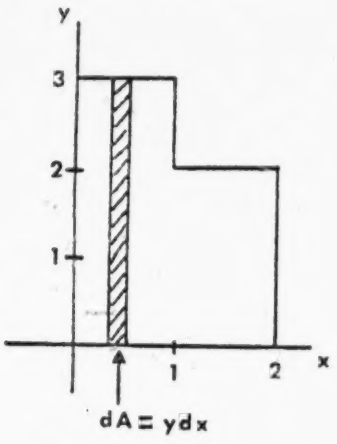
Another way to do this problem is to view this board as a continuous object and use the equivalent integral expressions for locating the center of mass. Thus,



$$x_{cm} = \frac{1}{M}\int x\ dm = \frac{\rho}{M}\int x\ dA = \frac{\rho}{M}\int xy\ dx$$

The value of y is:

$y = 3$ for $0 \leq x \leq 1$, and

$y = 2$ for $1 \leq x \leq 2$

Therefore,

$$x_{cm} = \frac{\rho}{M}\left[3\int_0^1 x\ dx + 2\int_1^2 x\ dx\right] = \frac{\rho}{5\rho}\left[\frac{3}{2}(x^2)\Big|_0^1 + \frac{2}{2}(x^2)\Big|_1^2\right]$$

$$= \frac{1}{5}\left[1.5 \times (1 - 0) + (4 - 1)\right] = \frac{1.5 + 3}{5} = 0.9 \text{ ft}$$

In a similar fashion

$$y_{cm} = \frac{1}{M}\int y\ dm = \frac{\rho}{M}\int yx\ dy$$

with:

$x = 2$ for $0 \leq y \leq 2$ and

$x = 1$ for $2 \leq y \leq 3$

Therefore,

$$y_{cm} = \frac{\rho}{5\rho}\left[2\int_0^2 y\ dy + \int_2^3 y\ dy\right] = \frac{1}{5}\left[\frac{2}{2}(y^2)\Big|_0^2 + \frac{1}{2}(y^2)\Big|_2^3\right]$$

$$= \frac{1}{5}\left[(4 - 0) + 0.5(9 - 4)\right] = \frac{4 + 2.5}{5} = 1.3 \text{ ft}$$

The latter method is, of course, more tedious. If, however, the piece of wood could not be divided into simple symmetrical parts (consider a semi-circle for example), the integral method would be the only one that could be used.

TRUE OR FALSE? The position of C_1 and C_2 in each square of wood in the first solution may be determined by symmetry considerations.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL The Momentum of a Particle

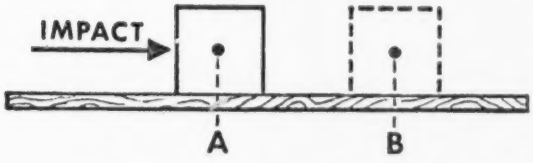
OBJECTIVE

To state and interpret the definition of momentum; to solve descriptive and numerical problems involving the momentum of particles with constant mass.

Although the word "momentum" appeared in the literature of classical physics subsequent to Newton's statement of his laws of motion, it is evident from his writings that he recognized the product of mass and velocity as an important physical entity. In his statement of the second law, he refers to what we now call momentum as the "quantity of motion" of a body. Since the word "quantity" is broad in meaning, the phrase "quantity of motion" is vague and unsatisfying.

Let's try to clarify its significance with the help of a simple example.

A steel block rests on a horizontal wood plank with its center of mass directly above point A. An experimenter, for reasons known only to himself, wishes to have the block moved to position B by permitting some other object moving toward the block from the left to collide with it. He has available to him only two things he can use: a hammer with a very massive head and a number of different cartridges together with the rifle that can fire them. He decides to strike the block with the hammer and, after a number of trials, find the *speed of impact* required with that particular hammer *mass* to successfully move the block of steel from A to B. In the second part of his experiment, he tries various bullet sizes and speeds until he finds one combination, say a .22 caliber slug with a definite muzzle velocity, which produces exactly the same motion of the block when allowed to strike it horizontally from the left. He now concludes that the hammer and the bullet had the same *quantity of motion* because they both produced the same motional change in the steel block. Thus, despite the crudity of the experiment and the possible hidden variables that may be present, he has explicitly defined quantity of motion in terms of the effect it has on another object.



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It is clear that both speed and mass must enter into a judgment of the quantity of motion possessed by the moving body. From the preceding example, it can be shown that quantity of motion, and the product of mass and velocity are equivalent. Using p to represent quantity of motion or *momentum*:

$$p_{hammer} = Mv \qquad \text{and} \qquad p_{bullet} = mV$$

in which we have used upper and lower case letters to symbolize large and small quantities, respectively.

Thus, *momentum is an entity* comprising a mass term and a velocity term in the form of a product. Two moving bodies (i.e., a massive hammer and a bullet of small mass) can be given the same momentum despite the mass disparity by giving them the correct velocities.

Since mass is a scalar and velocity is a vector quantity, the product is a vector, hence the definition of momentum should be written:

$$\vec{p} = m\vec{v}$$

In the British system, the unit for momentum is the slug-ft/sec. In the MKS system it is the kg-m/sec. In problem work, caution in unit conversions must be exercised to be sure that the dimensions of momentum are properly expressed.

All of the statements regarding the momentum of a particle apply equally well to real bodies when all the mass of the body is considered to be concentrated at the center of mass.

When solving the problems in this set, you will be asked to

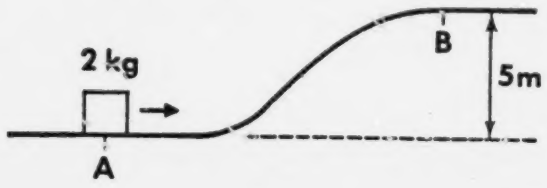
(a) determine the momentum of an object with given mass. In this problem, you will want to find the velocity of the body by applying the given data in the law of conservation of energy;

(b) solve a simple momentum problem involving unit conversions;

(c) determine momentum by first finding the velocity of a body from a knowledge of its kinetic energy.

PROBLEMS

1. A 2-kg block slides along the frictionless track shown in the figure. If the block's speed at point A is 10 m/sec, what is the momentum in kg-m/sec of the block at point B?

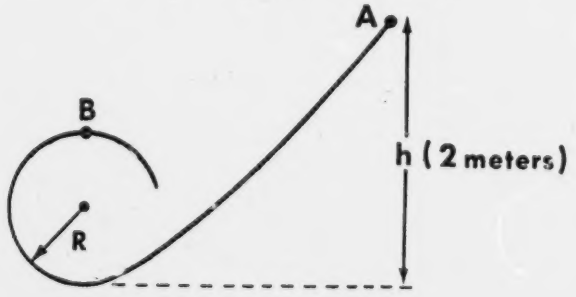


2. A 3200-lb automobile is heading north at a speed of 50 ft/sec. Its momentum is a vector directed north with a magnitude of

A. 160,000 lb-ft/sec
B. 160,000 slug-ft/sec
C. 5,000 slug-ft/sec
D. 5,000 lb-ft/sec

3. A 2-kg block slides with constant velocity down an inclined plane. The kinetic energy of the block is 16 joules. What is the magnitude of the block's momentum in kg-m/sec?

4. A particle of mass m = 2 kg slides down a track to enter an inside loop of radius R = 50 cm shown in the figure below. Without losing contact with the track at any time, it starts from rest at point A. What is its momentum at point B. Neglect friction.



INFORMATION PANEL

Momentum of a System of Particles

OBJECTIVE

To extend momentum concepts to the solution of problems involving systems of particles.

A careful analysis of the behavior of a system of particles shows that

> *the total momentum of a system of particles may be found by multiplying the total mass of the system by the velocity of the center of mass of the system.*

or

$$\vec{P} = M\vec{v}_{cm}$$

in which $\vec{P}$ = the total momentum of the system,
M = the total mass of the system,
$\vec{v}_{cm}$ = the velocity of the center of mass.

This problem section involves:

(a) a determination of the velocity of the center of mass of a system given the momentum of the system;

(b) finding the resultant momentum of a pair of particles by the method of vector addition;

(c) a determination of the direction of the net momentum of a given system of particles using vector methods.

5. Two particles of mass 2 kg and 3 kg respectively, are moving with a speed of 10 m/sec due east. A third particle of mass 2 kg is moving with a speed of 25 m/sec due north. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system of three particles.

A. 10.1 m/sec at 45° N of E
B. 20.2 m/sec at 37° N of E
C. 10.1 m/sec at 37° N of E
D. 20.2 m/sec at 45° N of E

6. Two bodies in a system have masses 8 kg and 12 kg and are moving with velocities 10 m/sec at 60° north of east and 5 m/sec at 30° south of east, respectively. The magnitude of momentum of the system is

A. 100 kg-m/sec
B. 140 kg-m/sec
C. 20 kg-m/sec
D. 70 kg-m/sec

7. A 2-kg particle moves due north at a speed of 1 m/sec. A second particle of mass 10 kg moves due east at a speed of 2 m/sec. What is the direction of the total momentum of the system?

A. 18° north of east
B. 12° north of east
C. 8° north of east
D. 6° north of east

8. A system of particles with masses of 8 kg and 12 kg has a total momentum of 100 kg-m/sec at 23° north of east. Determine the velocity of the center of mass of the system.

A. 100 m/sec; at 23° north of east
B. 140 m/sec; at due north
C. 20 m/sec; at 53° north of east
D. 5 m/sec; at 23° north of east

9. Two particles of mass m_1 = 2 kg and m_2 = 3 kg are moving with velocities of 10 m/sec due east and 20 m/sec due west, respectively. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system.

A. 8 m/sec; due west
B. 16 m/sec; due east
C. 16 m/sec; due west
D. 8 m/sec; due north

INFORMATION PANEL — The Second Law in Terms of Momentum

OBJECTIVE

To utilize Newton's second law expressed in momentum terms in interpreting certain physical situations and solving problems related to these situations.

Emphasis has been placed previously on the constraint that mass must be constant if the second law in the form

$$F = ma$$

is to be valid. In a number of cases, the mass of the system continually varies so that this equation can no longer be applied. For example, as a chemically propelled rocket moves, it burns fuel continuously so that its mass correspondingly decreases with time. Such problems are most easily handled by applying momentum considerations and, for this reason, it is important for you to be able to apply the second law in momentum terms with facility.

Newton's expression of the second law in Latin, when translated freely in modern terminology reads

> *The rate at which the momentum of a body changes is proportional to the resultant force acting on the body and takes place in the direction of the straight line in which the force acts.*

or

$$\vec{F} = \vec{dp}/dt$$

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If the mass is constant, the acceleration form of the second law is valid since

$$\vec{F} = d\vec{p}/dt = d(m\vec{v})/dt = m\, d\vec{v}/dt = m\vec{a}$$

You will find these formulations of the second law helpful in attacking the problems in this section.

10. The total mass of a system is 3 kg and the magnitude of the system's momentum is changing at the rate of 15 kg-m/sec^2. What is the magnitude of the net external force exerted on the system?

11. The total mass of a system is 100 gm, and the magnitude of the system's momentum is changing at the rate of 1000 gm-cm/sec^2. The magnitude of the acceleration of the center of mass of the system is

A. 1000 cm/sec^2
B. 10 cm/sec^2
C. 100,000 cm/sec^2
D. 98,000 cm/sec^2

12. The total mass of a system is 15 kg and the magnitude of the acceleration of its center of mass is 10 m/sec^2. What is the rate of change of the system's momentum?

INFORMATION PANEL Conservation of Momentum

OBJECTIVE

To apply the principle of conservation of momentum to the solution of typical problems in which this principle is found.

In the previous section of this segment of your work, you made use of Newton's second law in momentum terms, that is

$$\vec{F} = d\vec{p}/dt$$

in which $\vec{F}$ = the resultant force acting on the system, and $d\vec{p}/dt$ is the rate of change of momentum.

It follows directly from this that if the resultant force on the system is zero, then the rate of change of momentum of the system must also be zero, which in turn indicates that the momentum must remain constant.

$$\text{If } d\vec{P}/dt = 0, \text{ then } \vec{P} = \text{constant}.$$

In verbal form, this conclusion may be stated as follows:

> *In any system of interacting particles, the total vector momentum remains constant unless the system is acted on by an external net force.*

This statement implies that, although the momenta of individual particles may change from one moment to the next, their vector sum remains the same as long as no resultant force is applied.

Since momentum is a vector quantity, you must expect to use vector methods in summing up the momenta of particle systems, or in resolving a given particle momentum into components. The problems in this set entail your recognition of the fact that when the net external force is zero, momentum is conserved.

13. An 8-ton, open-top freight car is coasting at a speed of 5 ft/sec along a frictionless horizontal track. It suddenly begins to rain hard, the raindrops falling vertically with respect to ground. Assuming the car to be deep enough, so that the water does not spatter over the top of the car, what is the speed of the car after it has collected 4.5 tons of water?

14. A midshipman dives from the stern of a stationary rowboat. His mass is 70 kg and that of the rowboat 140 kg. The horizontal component of his velocity when his feet leave the boat is 3 m/sec relative to the water. What is the speed of the boat immediately after the dive?

15. A block of wood of mass M = 0.8 kg is suspended by a cord of negligible mass. A bullet of mass m = 4 gm is fired horizontally at the block with a muzzle velocity of 400 m/sec. The bullet remains embedded in the block. What is the speed with which the wood block (with bullet embedded) is set into motion?

16. Assume a rocket has an initial weight of 3000 tons and a weight of 2780 tons after the fuel is completely burned. Fuel is consumed at a rate of 2840 lb/sec. After what time interval in seconds does the rocket attain its maximum velocity?

17. Let $\vec{v}$ be the velocity of a rocket (mass m) relative to ground and $\vec{u}$ the velocity of the exhaust gases relative to the rocket. Newton's second law then becomes

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} - \vec{u}\frac{dm}{dt}$$

To which of the following does this equation reduce if the rocket is being held stationary on a test pad by bolts, which exert a force $\vec{F}_b$ on the rocket?

A. $m\vec{g} - \vec{F}_b = -\vec{u}\frac{dm}{dt}$

B. $\vec{F}_b = -\vec{u}\frac{dm}{dt}$

C. $m\vec{g} + \vec{F}_b = -\vec{u}\frac{dm}{dt}$

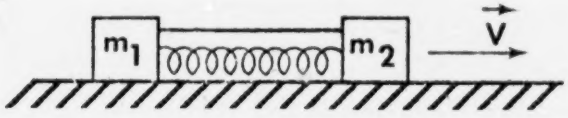
D. $m\vec{g} = -\vec{u}\frac{dm}{dt}$

18. Integrating the rocket equation given in problem 17 yields

$$\vec{v} = \vec{v}_o - \vec{u}\ln\left[\frac{m_o}{m}\right] + \vec{g}t$$

For the rocket and data given in problem 16, determine the maximum velocity of the rocket if the exhaust velocity of the gases relative to the rocket is 55,000 m/sec and the rocket started from rest.

19. Two masses are tied together with a compressed spring between the two as shown. The spring is not attached to either mass. The system slides on a frictionless table with a velocity $\vec{v}$. At some point the string is cut and the masses fly apart along the original line of motion. The velocities of mass m_1 and m_2 after release are $\vec{v}_1$ and $\vec{v}_2$, respectively. What was the impulse imparted to mass m_2?



A. $m_2(\vec{v}_2 - \vec{v}_1)$

B. $m_1(\vec{v}_1 - \vec{v}_2)$

C. $m_2(\vec{v}_2 - \vec{v})$

D. $m_2(\vec{v}_1 - \vec{v})$

20. For the system in problem 19, what is the impulse imparted to m_1?

A. $-m_1(\vec{v}_1 - \vec{v})$

B. $-m_2(\vec{v}_2 - \vec{v})$

C. $m_1(\vec{v} - \vec{v}_1)$

D. $m_1(\vec{v}_2 - \vec{v})$

21. For the system in problem 19, what is the momentum of the center of mass of the system after the string has been cut and the masses have attained their final velocities?

A. $m_2(\vec{v}_2 - \vec{v})$

B. $m_1(\vec{v}_1 - \vec{v})$

C. $(m_1 + m_2)\dfrac{\vec{v}_1 + \vec{v}_2}{2}$

D. $(m_1 + m_2)\vec{v}$

[a] CORRECT ANSWER: $2\sqrt{2}$ kg-m/sec

Since non-conservative (frictional) forces are absent, the total energy of the block is conserved. If the potential energy is taken to be zero at point A, we have

$$K_A = K_B + mgh \qquad (1)$$

with the subscripts A and B referring to points A and B, respectively. Since

$$K = \frac{mv^2}{2}$$

(1) becomes

$$\frac{mv_A^2}{2} = \frac{mv_B^2}{2} + mgh$$

Solving for v_B, we have

$$v_B = \sqrt{v_A^2 - 2gh}$$

and the momentum p_B at point B is given by

$$p_B = mv_B = m\sqrt{v_A^2 - 2gh}$$
$$= 2\sqrt{100 - 98} = 2\sqrt{2} \text{ kg-m/sec}$$

TRUE OR FALSE? The momentum of the block is independent of its position along the incline.

[b] CORRECT ANSWER: 15 nt

Newton's second law of motion can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt}$$

This shows that the force exerted on a body is equal to the time rate of change of its momentum. Hence, the magnitude of the force exerted on the given system is 15 kg-m/sec^2 = 15 nt.

TRUE OR FALSE? In this problem, $d\vec{p}/dt$ is equal to 15 kg-m/sec^2.

[a] CORRECT ANSWER: D

The total momentum $\vec{p}$ of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass,

$$\vec{p} = M\vec{v}_{cm}$$

Solving for $\vec{v}_{cm}$, we obtain

$$\vec{v}_{cm} = \frac{\vec{p}}{M} = \frac{100 \text{ kg-m/sec}}{(8 + 12) \text{ kg}} \ (23^\circ \text{ N of E}) = 5 \text{ m/sec } (23^\circ \text{ N of E})$$

[b] CORRECT ANSWER: 150 kg-m/sec^2

Newton's second law of motion can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F} = \text{net external force} \qquad (1)$$

This shows that the external force exerted on a system is equal to the time rate of change of its momentum. However, the net external force exerted on a system is also given by

$$\vec{F} = M\vec{a}_{cm} \qquad (2)$$

Where M is the total mass of the system and a_{cm} is the acceleration of its center of mass.

Equating equations (1) and (2) yields

$$\frac{dp}{dt} = M\vec{a}_{cm} = \text{the magnitude of the rate of change of momentum}$$

Substituting numerical data, we obtain

$$\frac{dp}{dt} = 15 \text{ kg} \times 10 \text{ m/sec}^2 = 150 \text{ kg-m/sec}^2$$

TRUE OR FALSE? The rate of change of the momentum of a system on which a net force acts is numerically equal to the rate of change of this force.

[a] CORRECT ANSWER: 1.99 m/sec

If v is the speed of the bullet and V the speed of the block with the bullet embedded in it, then by the conservation of momentum, we have

$$mv = (m + M)\, V \qquad (1)$$

Therefore, the required speed V is

$$V = \frac{m}{m + M}\, v \qquad (2)$$

Substitution of numerical values in equation (2) yields

$$V = \frac{4 \times 10^{-3}\ \text{kg}}{0.804\ \text{kg}} \times 400\ \text{m/sec}$$

$$= 1.99\ \text{m/sec}$$

TRUE OR FALSE? The magnitude of the force of gravity acting on the block and bullet is an important consideration in this solution.

[b] CORRECT ANSWER: 3.2 ft/sec

There are no external forces in the horizontal direction acting on the car-water system. Therefore, momentum is conserved. Thus,

$$m_i v_i = m_f v_f$$

and

$$v_f = \frac{m_i}{m_f}\, v_i = \frac{m_i g}{m_f g}\, v_i = \frac{8\ \text{tons}}{12.5\ \text{tons}} \times 5\ \text{ft/sec} = 3.2\ \text{ft/sec}$$

Note that, since the mass (or weight) of the system is involved in a ratio, no conversion to slugs (lb) is necessary.

TRUE OR FALSE? This problem could have been solved correctly by converting the weight of the car and the weight of the collected water to pounds before substituting the numbers.

[a] CORRECT ANSWER: C

The magnitude of the momentum is equal to the mass of the automobile times its speed. Since $w = mg$, $m = w/g$ and

$$mv = \frac{w}{g} v = \frac{3200 \text{ lb}}{32 \text{ ft/sec}^2} \times 50 \frac{\text{ft}}{\text{sec}} = 5000 \text{ slug-ft/sec}$$

[b] CORRECT ANSWER: 8.9 kg-m/sec

Since the frictional force is negligible, the system under consideration is conservative, and consequently the total energy of the point mass remains constant. If the potential energy is taken to be zero at the bottom of the loop, we have

$$mgh = mg(2R) + K_B \qquad (1)$$

Where K_B is the kinetic energy of the particle at point B. However,

$$K_B = \frac{P_B^2}{2m} \qquad P_B = \text{momentum of the particle at point B}$$

Therefore, equation (1) becomes

$$mgh = mg(2R) + \frac{P_B^2}{2m} \qquad (2)$$

Solving for P_B yields

$$P_B = \sqrt{2m^2g(h - 2R)}$$

$$= \sqrt{2 \times 4 \times 9.8 \times 1}$$

$$= 8.9 \text{ kg-m/sec}$$

TRUE OR FALSE? The momentum of the sliding particle is greater at point B than it was at point A.

[a] CORRECT ANSWER: A

The momentum $\vec{P}$ of the center of mass is equal to the sum of the individual momenta. The resultant momentum in the easterly direction has a magnitude given by

$$P_E = (2 \text{ kg})(10 \text{ m/sec}) + (3 \text{ kg})(10 \text{ m/sec})$$

$$= 50 \text{ kg-m/sec}$$

In the northerly direction, the momentum has a magnitude P_N:

$$P_N = (2 \text{ kg})(25 \text{ m/sec}) = 50 \text{ kg-m/sec}$$

From the vector diagram of $\vec{P}_E$ and $\vec{P}_N$, we can calculate total momentum $\vec{P}$,

$$P = \sqrt{P_E^2 + P_N^2} = 71 \text{ kg-m/sec}$$

$$\theta = \tan^{-1}(P_N/P_E) = \tan^{-1}1 = 45°$$

Finally, $\vec{V}_{cm} = \vec{P}/M$

$$V_{cm} = P/M = 71/(2 + 3 + 2) = 10.1 \text{ m/sec}$$

TRUE OR FALSE? The velocity of the center of mass of this system is determined by adding the individual particle velocities algebraically.

[a] CORRECT ANSWER: A

The total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of the center of mass,

$$\vec{P} = (m_1 + m_2)\, v_{cm} \qquad (1)$$

Where

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = m_1\vec{v}_1 + m_2\, \vec{v}_2 \qquad (2)$$

Where $\vec{v}_1$, $\vec{P}_1$, and $\vec{v}_2$, $\vec{P}_2$ are the velocities and momenta of the particles of mass m_1 and m_2, respectively. Let the easterly direction lie along the positive x-axis so that the westerly direction will lie along the negative x-axis. Hence $\vec{v}_1$ and $\vec{v}_2$ have components in the x-direction only. Therefore,

$$P = m_1v_1 - m_2v_2$$

$$= 2 \text{ kg} \times 10 \text{ m/sec} - 3 \text{ kg} \times 20 \text{ m/sec}$$

$$= -40 \text{ kg-m/sec}$$

The magnitude of the total momentum is 40 kg-m/sec and is due west.

From equation (1) we have

$$\vec{v}_{cm} = \frac{\vec{P}}{(m_1 + m_2)}$$

Substitution of the numerical values yields

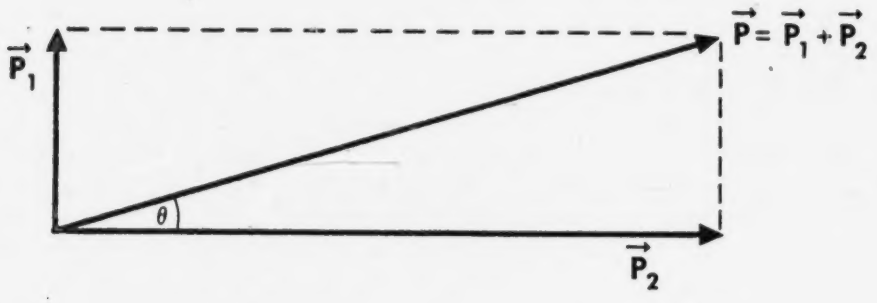
$$v_{cm} = \frac{40 \text{ kg-m/sec}}{(2 \text{ kg} + 3 \text{ kg})} = 8 \text{ m/sec}$$

The direction of v_{cm} is the same as that of the total momentum P, i.e., along the negative x-axis.

TRUE OR FALSE? The direction of the center-of-mass velocity vector is the same as that of the total momentum vector.

[a] CORRECT ANSWER: D

Introduce the notation $\vec{P}_1$ and $\vec{P}_2$ for the momenta of the 2-kg and 10-kg particles, respectively. We want to calculate the angle θ as shown in the diagram below.



From the geometry, we have

$$\tan\theta = P_1/P_2$$

substitute values of P_1 and P_2 to find

$$\tan\theta = (2 \text{ kg} \times 1 \text{ m/sec})/(10 \text{ kg} \times 2 \text{ m/sec})$$

$$= 0.1$$

Therefore,

$$\theta = \tan^{-1}0.1 \approx 6^\circ$$

[a] CORRECT ANSWER: C

The external forces on the rocket are those exerted by gravity and the bolts, both downward. Since the rocket is stationary, dv/dt is zero, and the "rocket equation",

$$\vec{F}_{ext} = m \frac{d\vec{v}}{dt} - \vec{u} \frac{dm}{dt}$$

reduces to

$$\vec{F}_{ext} = -\vec{u} \frac{dm}{dt}$$

Finally, the external force is given by

$$\vec{F}_{ext} = m\vec{g} + \vec{F}_b = -\vec{u} \frac{dm}{dt}$$

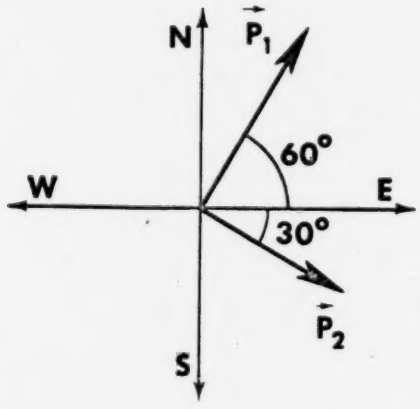
[b] CORRECT ANSWER: D

The whole system was moving with a velocity $\vec{v}$ before the string was cut. Hence, $\vec{v}$ is the initial velocity of the center of mass. Since the table is frictionless, the external force acting on the system, both before and after the string is cut, is zero. Therefore, the momentum (and velocity) of the system remain unchanged. The final velocity of the center of mass is $\vec{v}$ and the momentum is $(m_1 + m_2)\vec{v}$.

TRUE OR FALSE? The momentum of the system remains unchanged because all of the forces involved in this interaction are internal ones.

[a] CORRECT ANSWER: A

The momentum of a system is the vector sum of the individual momenta.



$\vec{p}_1$ = 8 kg × 10 m/sec = 80 kg-m/sec; at 60° N of E

$\vec{p}_2$ = 12 kg × 5 m/sec = 60 kg-m/sec; at 30° S of E

Using the fact that $\vec{p}_1$ and $\vec{p}_2$ form a right angle, we find

$$p = \sqrt{p_1^2 + p_2^2} = \sqrt{80^2 + 60^2} = 100 \text{ kg-m/sec}$$

[b] CORRECT ANSWER: B

For a system whose mass is constant we have

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

so

$$a = \frac{1}{m}\frac{dp}{dt} = \frac{1000 \text{ gm-cm/sec}^2}{100 \text{ gm}} = 10 \text{ cm/sec}^2$$

[a] CORRECT ANSWER: C

From the impulse-momentum principle we know that to find the impulse imparted to m_2, we must compute the change in the momentum of m_2. Thus,

$$\vec{J}_2 = \Delta\vec{p}_2 = \vec{p}_{2f} + \vec{p}_{2i} = m_2(\vec{v}_2 - \vec{v})$$

TRUE OR FALSE? Frictional forces must be considered when calculating the change in momentum of mass 2.

[b] CORRECT ANSWER: 155 seconds

The maximum velocity will occur at the instant the fuel has been consumed. Until then, the rocket accelerates; thereafter, it decelerates.

The rocket is losing mass (or weight) at a constant rate. Thus

$$\frac{d(mg)}{dt} = \frac{dw}{dt} = -2840 \text{ lb/sec}$$

or

$$\int_0^t dt = -\frac{1 \text{ sec}}{2840 \text{ lb}} \int_{w_i}^{w_f} dw$$

and

$$t = -\frac{1 \text{ sec}}{2840 \text{ lb}} (w_f - w_i) = -\frac{1 \text{ sec}}{2840 \text{ lb}} \left[(2780 - 3000) \text{ tons} \times 2000 \frac{\text{lb}}{\text{ton}}\right]$$

$$= 155 \text{ sec}$$

TRUE OR FALSE? According to the conditions given in the solution, this rocket must be traveling horizontally in a vacuum.

[a] CORRECT ANSWER: 8 kg-m/sec

The known quantity is kinetic energy K,

$$K = mv^2/2$$

where m is the mass of the block, and v is its speed.

Solving for v,

$$v = \sqrt{2k/m}$$

so the momentum, p, is given by

$$p = mv = m\sqrt{2k/m}$$

$$= \sqrt{2mk}$$

$$= 8 \text{ kg-m/sec}$$

[b] CORRECT ANSWER: 1.5 m/sec

The momentum of the system is zero before the dive. In the absence of an external force, momentum is conserved during the dive; therefore, the momentum of the system after the dive is also zero. We simply have to solve the equation $m_1 v_{1x} + m_2 v_{2x} = 0$ for v_{2x}. Thus,

$$v_{2x} = -\frac{m_1 v_{1x}}{m_2} = -\frac{70 \text{ kg} \times 3 \text{ m/sec}}{140 \text{ kg}} = -1.5 \text{ m/sec}$$

the minus sign indicating that $\vec{v}_{2x}$ is directed oppositely to $\vec{v}_{1x}$.

[a] CORRECT ANSWER: 2710 m/sec

With $v_o = 0$ and the upward as the positive direction in the given equation we obtain

$$v = u \ln\left[\frac{m_o}{m}\right] - gt$$

The maximum velocity is attained at t = 155 sec, at which time mg = 2780 tons. Now $m_o g = 3000$ tons and u = 55,000 m/sec, so

$$v_{max} = 55{,}000 \ln\left[\frac{3000}{2780}\right] - [9.8 \times 155] = 2710 \text{ m/sec}$$

TRUE OR FALSE? The weight of the rocket reaches its minimum value at the instant that the rocket attains its maximum speed.

[b] CORRECT ANSWER: B

The impulse imparted to m_1 is

$$\vec{J}_1 = \Delta\vec{p} = \vec{p}_{1f} - \vec{p}_{1i} = m_1(\vec{v}_{1f} - \vec{v}_{1i}) = m_1(\vec{v}_1 - \vec{v}) \qquad (1)$$

This expression, however, is not listed among the answer choices.

We may use Newton's third law of motion which states that the force exerted by m_1 on m_2 (with the spring as intermediary), is equal to the negative of the force exerted by m_2 on m_1. Since the forces also act for the same duration we have

$$\vec{J}_1 = \int_{t_i}^{t_f} \vec{F}_{12}\, dt = -\int_{t_i}^{t_f} \vec{F}_{21}\, dt = -\vec{J}_2$$

Using the result of the preceding problem, we obtain

$$\vec{J}_1 = -\vec{J}_2 = -m_2(\vec{v}_2 - \vec{v})$$